Compact Reactor

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Abstract. Weyl's Gauge Principle of 1929 has been used to establish Weyl's Quantum Principle (WQP) that requires that the Weyl scale factor should be unity. It has been shown that the WQP requires the following: quantum mechanics must be used to determine system states; the electrostatic potential must be non-singular and quantified; interactions between particles with different electric charges (i.e. electron and proton) do not obey Newton's Third Law at sub-nuclear separations, and nuclear particles may be much different than expected using the standard model. The above WQP requirements lead to a potential fusion reactor wherein deuterium nuclei are preferentially fused into helium nuclei. Because the deuterium nuclei are preferentially fused into helium nuclei at temperatures and energies lower than specified by the standard model there is no harmful radiation as a byproduct of this fusion process. Therefore, a reactor using this reaction does not need any shielding to contain such radiation. The energy released from each reaction and the absence of shielding makes the deuterium-plus-deuterium-to-helium (DDH) reactor very compact when compared to other reactors, both fission and fusion types. Moreover, the potential energy output per reactor weight and the absence of harmful radiation makes the DDH reactor an ideal candidate for space power. The logic is summarized by which the WQP requires the above conditions that make the prediction of DDH possible. The details of the DDH reaction will be presented along with the specifics of why the DDH reactor may be made to cause two deuterium nuclei to preferentially fuse to a helium nucleus. The presentation will also indicate the calculations needed to predict the reactor temperature as a function of fuel loading, reactor size, and desired output and will include the progress achieved to date.

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INTRODUCTION

Space travel needs a source of energy that is as compact as possible. Fusion energy may allow for this compactness if the temperature required for initiating the fusion reaction could be reduced and radioactive radiation could be avoided. The objective of this paper is to present a derivation of a non-singular electrostatic potential that leads to the prediction of a lower temperature fusion of two deuterium nuclei preferentially to a helium nucleus thereby avoiding all radioactive radiation.

NON-SINGULAR ELECTROSTATIC POTENTIAL

Weyl first proposed his scale factor in his 1918 attempt to embed electromagnetism into geometry (Weyl, 1918). Schrödinger noticed that for a large number of systems satisfying the Bohr-Sommerfeld quantization conditions, the exponent of the non-integrable Weyl scale factor:

$$\ell = \ell_0 \exp\left[\frac{1}{\gamma}\int \phi_j dx^j\right], \tag{1}$$

became quantized (Schrödinger, 1922). He also showed that, if the unit of quantization was taken to be imaginary with a magnitude of Planck's constant, then the Weyl factor would be unity. London showed that the Weyl scale factor was proportional to the Schrödinger wave function and that, if one knew the gauge potentials appearing in the exponent of the Weyl factor and asked what paths are allowed if the exponent were to remain quantized, the paths allowed were those given by Schrödinger wave mechanics (London, 1927). Though London's reinterpretation was tentative, Weyl seized upon it and presented a formulation that was complete and went further to propose his scale factor as a principle since electromagnetism could be *derived* from the gauge potentials (Weyl, 1929). The analysis would include the fact that Weyl introduced a special case of Nother's theorem, which displayed the analogy between energy-momentum and electromagnetic conservation laws and, thereby, made the result familiar to physicists in the context of field theory.

Many scientists have engaged in gauge theory research as a result of Weyl's initial work. These researchers have given numerous invaluable contributions to the field. However, the synopsis above is sufficient to display that the basis of quantum gauge theory began with the concept of quantized motion. London showed that the solutions to Schrödinger's wave equation determined the paths that were allowed, while Weyl's scale factor remained unity and its exponent quantized. This provided a description of quantized paths or motion.

The exponent of the scale factor may be thought of as having three parts: an integrand (one or more of the respective gauge potentials), a path (which is given by the differentials) and an integral value (the result of integrating the integrand over the path.) Thus, while London showed that the equations of quantum mechanics gave the paths allowed by a quantized exponent, provided the gauge potentials were known, another question might be asked of the exponent with equal expectations with regard to the descriptions of physical phenomena.

As may be recalled, Einstein's objection to Weyl's proposal of using his gauge geometry as a unifying theory of electromagnetism and gravity was primarily based upon the fact that the Weyl manifold was non-integrable. Einstein argued that such a path dependent manifold could not hope to describe such phenomena as atomic states that experimentally were determined to be independent of their history. What Schrödinger recognized and London proved was that the Weyl Quantum Principle, by setting the scale calibration to unity, filtered out gauges that were not integrable in order to leave only those that were integrable. Therefore, those paths, or states, that London showed resulted from imposing Weyl's Quantum Principle, that is, the paths determined by quantum mechanics, were not subjected to the history dependence contained in Einstein's objection. It is important to note that imposing Weyl's Quantum Principle did not select a single path independent state; rather, it stated that an infinite number of states were equally path independent. The states are separated from each other because each one has a different quantum number. Where Einstein may have expected to find a single integrable manifold, Weyl's Quantum Principle produces an infinite number of them.

Before presenting an additional question that Weyl's Principle may address, it must first be pointed out that all known particles with electric charge exhibit the property of quantization of electrostatic charge. It is these quantized potentials and the interactions of a particle with electromagnetic fields that establishes the particle's identity. If this identity is to persist in time and through movement in space, the identifying gauge potentials must be independent of the motion. This is the same question that concerned Einstein about atomic states. The atomic states reflect the interaction of two, or more, particles. Here the question concerns the identity of a particle. In the atomic states the interacting particles do not change their gauge properties during the motion, or time, involved in this interaction. London found that the Weyl Principle pointed to an infinite number of states that were independent of the path. The question to be asked now is, "Can the Weyl Quantum Principle also point to the gauge potentials that may be possessed by particles such that these properties are independent of the path?"

Weyl's Principle demands that the scale, or gauge, Equation (1), of the manifold be unity with $\ell = \ell_o$ for any path. Obviously, then:

$$e^{\int \phi_j dx^j} = 1 = e^{a+ib} = e^a(\cos b + i \sin b), \tag{2}$$

for which $a = 0$ and $b = \cos^{-1}(1) = \sin^{-1}(0) = 2\pi N$ with $N = 0, \pm 1, \pm 2, \ldots$. Thus, the gauge potentials sought must satisfy:

$$\int \phi_j dx^j = i2\pi N, \quad \text{where } \varphi_j \equiv \frac{1}{2}\ln\left(\frac{\partial f}{\partial x^j}\right), \tag{3}$$

and the i on the right hand side indicates an imaginary value, f is the gauge function and the summation convention is to be applied to the j's. Here N has been used to distinguish the gauge potential quantum number from the previous orbital, or path, quantum number, n. If the gauge potentials are to be independent of the dx^j then all of the path elements may be chosen to be zero except a single arbitrary dx^j. Therefore, the gauge potentials must satisfy the condition that:

$$\int \phi_j dx^j = i2\pi N_j , \tag{4}$$

where now the summation convention is not used and each gauge potential is quantized by the value N_j. When ϕ_0 is identified with the electrostatic potential, as is done in quantum mechanics, the quantum number, N_0, quantizes the charge of the electrostatic potential. The components of the gauge vector potential, (ϕ_1, ϕ_2, ϕ_3), must also be quantized. The four components of the gauge potential must then be quantized as ($N_0\phi_0$, $N_1\phi_1$, $N_2\phi_2$, $N_3\phi_3$). Therefore, the same quantum condition which Schrödinger, London and Weyl used to quantize the paths allowed by a unity scale factor also quantizes the gauge potentials allowed for particles!

Any potential $\phi_{j=} N_j\phi_j$ allowed by the quantum condition and the gauge fields which come from the gauge potentials, that is:

$$F_{jk} \equiv \frac{\partial \phi_j}{\partial x^k} - \frac{\partial \phi_k}{\partial x^j} , \tag{5}$$

must also satisfy Maxwell's electromagnetic field equations. It has been shown (Williams, 2001) that Maxwell's equations produce a non-singular electrostatic potential:

$$f_r = \frac{k}{r} e^{-\frac{\lambda_N}{r}} \tag{6}$$

where the subscript in the exponent indicates that the exponent depends upon the particle's gauge potential quantum numbers, as $\lambda_N = K\lambda_0$, and may be, therefore, different for different particles. Indeed the K is dependent upon the gauge potential quantum numbers, N_j. If the gauge potentials do not possess quantum numbers, that is when the scale factor is path dependent as Einstein supposed, then K=0 and the potential of Equation (6) reduces to the classical electrostatic potential. The k appearing in the numerator also depends upon the particle as it is the usual electrostatic constant $(Z_1e_1Z_2e_2)/4\pi\varepsilon_o$.

NUCLEAR INTERACTIONS

The non-singular potential of Equation (6) indicates that interactions between two particles for which $\lambda_1 \neq \lambda_2$ will not satisfy Newton's Third Law. Therefore any investigation into interactions between particles with these non-singular potentials must do so without imposing Newton's Third Law.

Motion OF and ABOUT the Center of Mass

It has been shown that, while it is easier to study multiple body problems when Newton's Third Law holds, it is not mandatory (Williams, 2001). The two-body problem may be written in terms of the motion of each body or as the motion OF the center of mass and motion ABOUT the center of mass. While it may be tempting to use the motion of each body it is easier to compare with classical motions when using the motions OF and ABOUT the center of mass. It is then easy to see the conditions for which Newton's Third Law holds for when it holds there will be no motion OF the center of mass.

Newtonian Motion OF and ABOUT the center of mass

An investigation of the equations of motion may start with the well-known change of coordinates that allows one to analyze the motion in terms of the center of mass,

$$\begin{aligned} \overline{R} &= \frac{m_1\overline{r}_1 + m_2\overline{r}_2}{m_1 + m_2}, \\ \overline{r} &= \overline{r}_1 - \overline{r}_2, \end{aligned} \tag{7}$$

where the small *r*'s represent vectors to each body, or between the bodies, and the capital *R* represents the vector to the center of mass of the two bodies. Of course the inverse transformations associated with the transformation contained in Equations (7) exist and are the usual ones associated with the center of mass (COM) approach.

By using the COM approach and writing the low velocity limit of the equations of motion (EOM), we find:

$$\begin{aligned} m_1\ddot{r}_1\hat{r}_1 &= \overline{F}_1^{\,i} + \overline{F}_1^{\,e}, \\ m_2\ddot{r}_2\hat{r}_2 &= \overline{F}_2^{\,i} + \overline{F}_2^{\,e}, \end{aligned} \tag{8}$$

where the superscripts *i* and *e* represent internal and external forces respectively and the ^ denotes a unit vector. By using the standard definition for the total mass, $M = m_1+m_2$, and the reduced mass, $\mu=(m_1m_2)/M$, and setting all external forces to zero, Equations (8) may be put into the form:

$$\begin{aligned} M\ddot{R}\hat{R} &= \overline{F}_1^{i} + \overline{F}_2^{i}, \\ \mu\ddot{r}\hat{r} &= \frac{\mu}{m_1}\overline{F}_1^{i} - \frac{\mu}{m_2}\overline{F}_2^{i}. \end{aligned} \tag{9}$$

The EOM, Equations (9), display the effect of Newton's Third Law on the two-body problem. Should Newton's Third Law hold in both magnitude and direction then the first equation shows that the force on the COM vanishes while the two forces, which must remain separate when Newton's Third Law does not hold, becomes a single-force statement without any reference to the mass of the bodies. Further, the first equation gives the motion *OF* the COM while the second equation gives the motion *ABOUT* the COM.

The force on body one due to the presence of body two is:

$$\overline{F}_1^{i} = -Z_1 e\overline{\nabla}V_2^{i} = -\left(\frac{Z_1Z_2e^2}{4\pi\varepsilon_0}\right)\frac{1}{r^2}\left(1-\frac{\lambda_2}{r}\right)e^{\left(-\frac{\lambda_2}{r}\right)}\hat{r}, \tag{10}$$

while the force on body two due to the presence of body one is:

$$\overline{F}_2^{i} = -Z_2 e\overline{\nabla}V_1^{i} = \left(\frac{Z_1Z_2e^2}{4\pi\varepsilon_0}\right)\frac{1}{r^2}\left(1-\frac{\lambda_1}{r}\right)e^{\left(-\frac{\lambda_1}{r}\right)}\hat{r}. \tag{11}$$

Equations (10) and (11) exhibit the property that at large separations they approximately obey Newton's Third Law, but as the separation approaches the larger of the λ's they begin to severely depart from Newton's Third Law. Therefore, these two forces cannot be combined into a single central force as is done in classical mechanics, nor can their potentials be combined into a single potential.

By using the equations of motion, Equations (9), with the force laws, Equations (10) and (11), the Conservation of Energy, and transferring to the cylindrical coordinates typical of motion for central forces, it may be shown that the energy, which is a constant of the low velocity motion, becomes:

$$E = K + V + K_c + V_c, \tag{12}$$

where $K + V$ is the energy *ABOUT* the COM and $K_c + V_c$ is the energy *OF* the COM. In Equation (12), the parts are given by:

$$\begin{aligned}
K &= \frac{\mu k}{2r}\left[\left(1-\frac{\lambda_1}{r}\right)\frac{e^{-\frac{\lambda_1}{r}}}{m_2} + \left(1-\frac{\lambda_2}{r}\right)\frac{e^{-\frac{\lambda_2}{r}}}{m_1}\right] \\
V &= -\frac{\mu k}{r}\left[\frac{e^{-\frac{\lambda_1}{r}}}{m_2} + \frac{e^{-\frac{\lambda_2}{r}}}{m_1}\right] \\
K_c &= \frac{kR}{2r^2}\left[\left(1-\frac{\lambda_1}{r}\right)e^{-\frac{\lambda_1}{r}} - \left(1-\frac{\lambda_2}{r}\right)e^{-\frac{\lambda_2}{r}}\right] \\
V_c &= -\frac{k}{r}\left[e^{-\frac{\lambda_1}{r}} - e^{-\frac{\lambda_2}{r}}\right],
\end{aligned} \tag{13}$$

where $k=(Z_1Z_2e^2)/(4\pi\varepsilon_0)$.

Quantum Mechanical Motion OF and ABOUT the Center of Mass

Both non-relativistic and relativistic quantum mechanics may also be developed without the assumption of Newton's Third Law. Some surprising results occur as the result of this development.

Non-Relativistic Quantum Mechanical Motion OF and ABOUT the Center of Mass

An expanded Schrödinger-like wave equation may be developed using similar assumptions that Schrödinger used (Schrödinger, 1926) except that Newton's Third Law is not assumed nor applied (Williams, 2001). When this is done, and the required linear behavior and free system limit is met, the resulting wave equation ends up being:

$$\begin{aligned}
&-\frac{\hbar^2}{\mu}\frac{\partial^2\psi(x,X,t)}{\partial x^2} + V(x,t)\psi(x,X,t) - \frac{\hbar^2}{2M}\frac{\partial^2\psi(x,X,t)}{\partial X^2} \\
&+V_c(x,t)\psi(x,X,t) = i\hbar\frac{\partial\psi(x,X,t)}{\partial t}.
\end{aligned} \tag{14}$$

The time-independent, expanded wave equation in cylindrical coordinates is then:

$$\begin{aligned}
&-\frac{\hbar^2}{2\mu}\frac{\partial^2\psi(\bar{r},\bar{R})}{\partial\bar{r}^2} + V(\bar{r},\bar{R})\psi(\bar{r},\bar{R}) - \frac{\hbar^2}{2M}\frac{\partial^2\psi(\bar{r},\bar{R})}{\partial\bar{R}^2} \\
&+V_c(\bar{r},\bar{R})\psi(\bar{r},\bar{R}) = E\psi(\bar{r},\bar{R}),
\end{aligned} \tag{15}$$

where the potentials are those in Equations (13).

Relativistic equations may also be developed without the imposition of Newton's Third Law through a long, laborious process that is virtually the same as the process used in standard relativistic quantum mechanics (Williams, 2001). Defining $B^{\beta}{}_{\theta\mu\nu}=\partial_{\theta\mu}A^{T\beta}{}_{\nu}-\partial_{\theta\nu}A^{T\beta}{}_{\mu}$ the Klein-Gordon equation with fields for the motion OF and ABOUT the center of mass for two particles may be written as:

$$[(i\partial_{\theta\mu}I^{\theta}-e_{\theta}A^{T\theta}_{\mu})(i\partial^{\mu}_{\theta}I^{\theta}-e_{\theta}A^{T\theta\mu})-(m_{\theta}I^{\theta})^{2}-i\sigma^{\mu\nu}e_{\beta}B^{\beta}_{\theta\mu\nu}I^{\theta}]\psi(x)=0, \quad (16)$$

with the usual definition for $A^{T\beta}{}_{\mu}$ and I^{θ} is the 1 by 2 identity matrix. The $B^{\beta}{}_{\theta\mu\nu}$ are defined as are the Yang-Mills fields.

Three-body problems may be determined by defining $D^{\eta}{}_{\theta\mu\nu}=\partial_{\theta\mu}C^{\eta}{}_{\nu}-\partial_{\theta\nu}C^{\eta}{}_{\mu}$ with the definition $C^{\theta}{}_{\mu}=A^{\beta}{}_{\mu}+A^{\gamma}{}_{\mu}$, where θ, β, and γ are cyclic. Equation (16) becomes:

$$\left[\left(i\partial_{\theta\mu}I^{\theta}-e_{\theta}C^{\theta}_{\mu}\right)\left(i\partial^{\mu}_{\theta}I^{\theta}-e_{\theta}C^{\theta\mu}\right)-\left(m_{\theta}I^{\theta}\right)-i\sigma^{\mu\nu}e_{\eta}D^{\eta}_{\theta\mu\nu}I^{\theta}\right]\psi(x)=0 \quad (17)$$

where θ, β, and η range from 1 to 3 and θ, β, and η are cyclic and the I^{θ} is the 1 by 3 identity matrix.

Equation (17) represents the Klein-Gordon equation for three particles when one particle is different from the other two. However, for three like particles it remains valid, but some terms vanish due to the equal and opposite forces. Also, Equation (17) is the equation describing the strong nuclear forces that bind two protons to a single electron to form the deuterium nucleus. Of course this occurs when the separation of all three particles is approximately of nuclear separations. It may also be seen that the three-body system of two protons in orbit around a single electron is a symmetrical system wherein the COM has no motion.

COMPACT FUSION REACTOR

The suggestion that a neutron consists of a proton in orbit around an electron described by Equation (16) is very different from the standard nuclear model and raises many questions such as, "Isn't this a violation of the Heisenberg uncertainty principle?" and "Doesn't such motion violate spin conservation?" These questions have already been answered (Williams, 1983). However, if a neutron is a proton in orbit around an electron, then an additional orbiting proton leads to a deuterium nucleus that provides for a very different prediction of a fusion reactor.

Each deuteron has a magnetic moment whose spin axis is normal to the plane of the three particles and the axis of the deuterons may be aligned end to end by the use of a magnetic field. Once this preconditioning has been done, if the deuterons are nudged together (see Figure 1), the long range repulsive interaction between protons cause the protons of the approaching deuterons to stay as far from each other as far as possible. The protons, therefore, self align to establish the minimum threshold energy for fusion.

On the other hand, the electrons repel other electrons though they may be attracted to protons. A very stable configuration may be obtained by the fusion of two deuterons. In this stable configuration the four protons are in orbit in a plane about an orbit spin axis while the two electrons are located on the spin axis equal distances above and below the plane of the protons orbit (see Figure 2).

The forces of the interactions between all six particles of the two deuterons have been written down and studied using a Runga-Kutta integration method in a spreadsheet. This study showed that this alignment of the deuterium nuclei spin axis provides a much lower threshold for fusion than all other orientations. Further, the deuterium nuclei in this orientation preferentially fuse to the helium nucleus without emitting any particle radiation.

However, since the forces and the resulting potentials are transcendental equations, they have not yet been analytically integrated and must be submitted to computer solution to determine the numerical value of the fusion threshold for different methods of nudging the deuterons together. Further, in order to complete the study of the deuterium-deuterium to helium reaction the relativistic solutions of Equations (17) must be obtained.

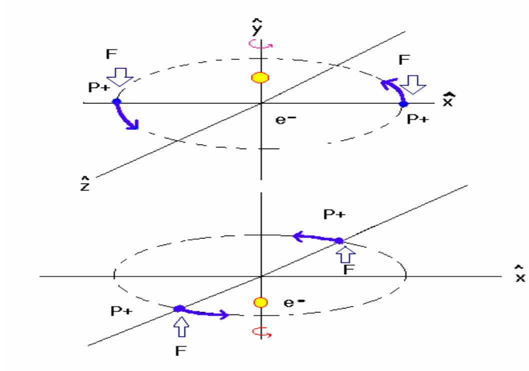

FIGURE 1. Two Deuterium Nuclei Being Nudged Together.

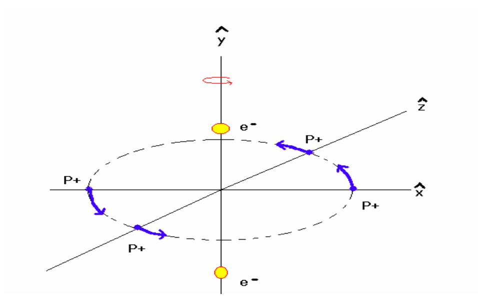

FIGURE 2. Helium Nucleus Formed From Two Deuterium Nuclei.

CONCLUSIONS

Weyl's Gauge Principle and the restriction to a unity scale factor not only require the electromagnetic gauge fields, but also require that the electrostatic potential be quantized and non-singular. The non-singular character of the potentials violates Newton's Third Law of equal and opposite forces. However, when quantum mechanical procedures are applied in the absence of Newton's Third Law the relativistic quantum mechanical equations of the motion OF and ABOUT the center of mass contain the Yang-Mills fields and the SU3 group fields of the standard model. Thus, while the new view and approach are completely different from the standard model, the standard model is not violated for its equations are contained within the new view. Plus, the new view adds predictions not possible in the standard model.

The equations of motion for the center of mass for the two-particle system of a proton and an electron shows that a bound orbit of the proton around the electron exists at nuclear separations (Williams, 2001). The center of mass of the neutron is bound inside a positive energy well from which it may escape by quantum tunneling. The rate of neutron disintegration may be calculated and shown to compare with the experimental half-life.

At nuclear separations another proton may be added to the neutron to form the deuterium nucleus with two protons in orbit around a single electron. This configuration for the deuterium nucleus shows that should one deuterium nucleus approach another at high energy and with a random orientation with respect to the first deuterium there will be very high threshold for fusion and an high probability that a tritium nucleus, wherein three protons orbits around a single electron, will be formed and a neutron released. However, when the spin axis of the two deuterium nuclei are aligned and the nuclei are placed so that their spin axis are aligned the natural tendency of protons long range repulsion causes the two spinning deuterium nuclei to lock into phase with each other where the lines between the protons within each deuterium nucleus are at ninety degrees to each other.

One means of holding two deuterium nuclei in such an orientation might be to place them into the crystal lattice of a metal under a high magnetic field that locks up their spin axis in the correct orientation. The crystal structure should be such that each deuterium nucleus should have a clear view of a neighboring deuterium nucleus along it spin axis without another nucleus in the path. Temperature vibrations such as to provide motion of the deuterium nuclei toward each other will set up the quantum tunneling that will result in the two deuterium nuclei preferentially fusing into a helium nucleus without any attendant radiation.

NOMENCLATURE

l = vector length (m)
ϕ_0 = gauge potential component (V)
F_{ij} = electromagnetogravitic field tensor (V/m)
f_r = electrostatic gauge potential (V)
m_i = mass (kg)
e = unit of charge (coul)
Z^i = atomic number (unitless)
F_i = force on particle I due to other particles present (N)
K = kinetic energy (N-m)
V = potential energy (N-m)
K_c = kinetic energy of the center of mass (N-m)
V_c = potential energy of the center of mass (N-m)
ε = permittivity (farad/m)
λ = radius of maximum magnitude of the potential (m)
$A^{T\beta}{}_{\mu}$ = transpose of electromagnetic vector potential (V)
$B^{\beta}{}_{\theta\mu\nu}$ = Yang-Mills fields (V/m$^{2)}$
$C^{\theta}{}_{\mu}$ = three-body vector potential (V)
$D^{\eta}{}_{\theta\mu\nu}$ = three-body strong nuclear field (V/m^2)

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