CHAPTER 2 NEW THEORETICAL FUNDAMENTALS

The Dynamic Theory uses a different viewpoint, or approach, to present a description of physical phenomena. Therefore the first criterion that it must meet is that it must not be in conflict with existing theories in a field of physics where the existing theory gives an adequate and accurate description. To show that the Dynamic Theory meets this criterion, this section will present the adopted laws and then proceed to show how the fundamental principles of existing theories may be obtained from these laws.

A. General Laws

In the following development physical concepts are necessary, as are symbols for these concepts. Because this development will merge certain thermodynamic conceptualizations into mechanics, a notational dilemma must be faced. On the one hand, it is desired to preserve the thermodynamic conceptualization by using familiar symbols from that theory. On the other hand, descriptions of mechanical systems are also sought. The formulism then looks either like thermodynamics with familiar thermodynamic quantities replaced by mechanical quantities, or it looks like mechanics into which thermodynamic quantities intruded. In either case there is danger of confusion. One could avoid the dilemma by choosing entirely different symbols for the variables of the theory. But then the whole takes on an artificially abstract character. Since the purpose of this formulation is to bring out the power of the thermodynamic conceptualization, it was decided to use the suggestiveness of the thermodynamic or mechanical symbols whenever convenient; the reader is asked to keep an open mind and not make premature association with the symbols used.

2.1 First Law.

The concept of conservation of energy is fundamental to all branches of physics and therefore represents a logical beginning for a generalized theory. Therefore, in terms of generalized coordinates or independent variables, the notion of work, or mechanical energy, is considered linear forms of the type

$$\_W = F_i(q^1,...,q^n,u^1,...,u^n)\, dq^i \quad (i = 1,2,...,n),$$

where the forces F_i may be functions of the velocities ($dq^i/dt = u^i$) as well as the coordinates q^i and the summation convention is used. The inclusion of velocities in forces reflects the belief that forces should depend upon the velocities. This will become clearer when these work terms are included in the first law.

The line integral $\int_c F_i \, dq^i$ then represents the work done along the path C by the generalized forces.

A system may acquire energy by other means in addition to the work terms; such energy acquisition is denoted dE.

The system energy, which represents the energy possessed by the system, is considered to be

$$U(q^1,\ldots,q^n, u^1,\ldots,u^n).$$

dU will be assumed to be a perfect differential.

With these concepts, then the generalized Law of Conservation of Energy, which is adopted as the first law of the Dynamic Theory, has the form

$$\begin{aligned} \_E &= dU - \_W \\ &= dU - F_i \, dq^i \quad (i = 1,\ldots,n). \end{aligned} \tag{2.1}$$

Positive *dE* is taken as energy added to the system by means other than through the work terms and F_i is taken as the component of the generalized force acting on the system which caused displacement dq^i.

In the First Law the dimensionality is n + 1 and is determined by the system considered. There is no limitation on the quantity or type of variables that may be used. However, in this presentation and in practice, it will be beneficial to place restrictions upon the type and number of allowed work terms. A system with only one work term, which is the pdv expansion work of classical thermodynamics, will be called a "thermodynamic" system and the dimensionality will be two. A system with three or less fdx work terms will be called a "mechanical" system with the appropriate dimensionality. Obviously, if there are three mechanical work terms, the dimensionality will be four. A system with a combination of thermodynamic and mechanical work terms will be considered later.

In an infinitesimal transformation, the First Law is equivalent to the statement that the differential

$$dU = \_E + F_i \, dq^i$$

is exact. That is, there exists a function U whose differential is dU; or the integral $\int dU$ is independent of the path of the integration and depends only on the limits of integration. This condition is not shared by $\int dE$ or $\int đW$. The path dependence of $\int đW$ is another reason that the generalized forces are assumed to be functions of velocity as well as position. In Newtonian mechanics forces are usually assumed to be dependent on position only so that the simplicity of path independence may be used. Though even in Newtonian mechanics certain forces are taken as velocity dependent. Friction forces are an example.

This statement of the generalized First Law is consistent with the First Law of thermodynamics in that if there is only one generalized force, which is taken to be the pressure, and one generalized coordinate, the volume, then Eqn. (2.1) becomes

$$\_Q = \_E = dU + Pdv$$

where F = -P with the convention that work of expansion is work done by the system on its surroundings. Here the system energy, U, is the thermodynamical internal energy. There should then be no confusion when Cartheodory's statement of the second law is applied to this thermodynamic system. However, when considering the application of generalizations of the classical thermodynamic laws to mechanical systems some confusion may be expected. During the initial portion of this development, it is desired to demonstrate the applicability of the generalized laws to mechanical systems. Therefore, it may help avoid confusion to think of the generalized coordinates of a mechanical system as the space coordinates of a mass point. Obviously, there exists systems in nature that may be considered to consist of a continuous distribution of mass points. Such a system may be thought of as a composite system of an infinite number of subsystems and, therefore, involve an infinite number of "generalized coordinates," or "degrees of freedom." However, just as in classical mechanics, we may later make the transition from mass points to matter in bulk; then the generalized coordinates, q^i, used here may better be termed independent variables.

To explore some of the consequences of the exactness of dU, consider a system whose variables are F, q and u. The existence of the state function U, or an equation of state, means that any pair of these three parameters may be chosen to be the independent variables that completely specify the system. For example consider U = U(F,q) then

$$dU = \left[\frac{\partial U}{\partial F}\right]_q dF + \left[\frac{\partial U}{\partial q}\right]_F dq.$$

The requirement that dU be exact immediately leads to the result

$$\frac{\partial}{\partial q}\left[\left[\frac{\partial U}{\partial F}\right]_q\right]_F = \frac{\partial}{\partial F}\left[\left[\frac{\partial U}{\partial q}\right]_F\right]_q.$$

The "energy capacity" of a system at the position q with dq = 0 may be defined as

$$C_q = \left[\frac{\_E}{\_u}\right]_q = \left[\frac{\partial U}{\partial u}\right]_{q'}$$

and the "energy capacity" of a system under a constant force is defined as

$$C_F = \left[\frac{\_E}{\_u}\right]_F = \left[\frac{\partial U}{\partial u}\right]_{F'}.$$

2.2 Second Law.

There are processes that satisfy the First Law but are not observed in nature. The purpose of the dynamic second law is to incorporate such experimental facts into the model of dynamics.

The statement of the Second Law is made using the axiomatic statement provided by the Greek mathematician Caratheodory, who presented an axiomatic development of the Second Law of thermodynamics that may be applied to a system of any number of variables. The Second Law may then be stated as follows:

> In the neighborhood (however close) of any equilibrium state of a system of any number of dynamic coordinates, there exist states that cannot be reached by reversible E - conservative (dE = 0) processes.

When the variables are thermodynamic variables, the E-conservative processes are known as adiabatic processes.

A reversible process is one that is performed in such a way that, at the conclusion of the process, both the system and the local surroundings may be restored to their initial states without producing any change in the rest of the universe.

Consider a system whose independent coordinates are a generalized displacement denoted q, a generalized velocity u (with u = dq/dt), and a generalized force F. It can be shown that the E-conservative curve comprising all equilibrium states accessible from the initial state, i, may be expressed by $\sigma(u,q)$ = constant, where σ represents some as yet undetermined function. Curves corresponding to other initial states would be represented by different values of the constant.

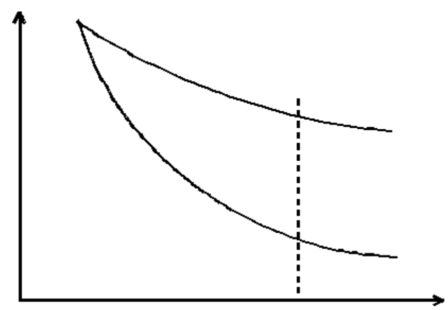

Figure 1. If two reversible E-conservative curves could intersect it would be possible to violate the Second Law by performing the cycle i, f_1, f_2, i.

Reversible E-conservative curves cannot intersect, for if they did, it would be possible, as shown in Figure 1, to proceed from an initial equilibrium state i, at the point of intersection, to two different final states f_1 and f_2, having the same q, along reversible E-conservative paths, which is not allowed by the Second Law.

When the system can be described with only two independent variables, such as on the E-conservative curve, then if these variables are q and u and F is a generalized force,

$$\_E = dU - Fdq.$$

Regarding U = U(q,u), then

$$\_E = \left[\frac{\partial U}{\partial u}\right]_q du + \left[\left[\frac{\partial U}{\partial q}\right]_u - F\right] dq$$

where all quantities on the right-hand side are functions of u and q.

An E-conservative process for this system is

$$\left[\frac{\partial U}{\partial u}\right]_q du + \left[\left[\frac{\partial U}{\partial q}\right]_u - F\right] dq = 0.$$

Solving for du/dq yields

$$\frac{du}{dq} = \frac{-\left[\left[\frac{\partial U}{\partial q}\right]_u - F\right]}{\left[\frac{\partial U}{\partial u}\right]_q}. \tag{2.2}$$

Figure 2. The First Law fills the (u,q) space with slopes. The σ curves represent the solution curves whose tangents are the required slopes. The Second Law requires that these curves do not intersect.

The right hand member is a function of u and q, and therefore, the derivative du/dq, representing the slope of a E-conservative curve on a (u,q) diagram, is known at all points. Equation (2.2) has therefore a solution consisting of a family of curves, see Figure 2, and the curve through any one point may be written

$$\sigma = \sigma(u,q) = constant.$$

A set of curves is obtained when different values are assigned to the constant. The existence of the family of curves σ(u,q) = constant, generated by Eqn. (2.2) representing reversible E-conservative processes, follows from the fact that there are only two independent variables and not from any law of physics. Thus it can be seen that the First Law may be satisfied by any of these σ = constant curves. The Second Law requires that these curves do not intersect. Therefore the Second Law, together with the First law, leads to the conclusion that through any arbitrary initial-state point, all reversible E-conservative processes lie on a curve, and E-conservative curves through other initial states determine a family of non-intersecting curves.

To see the results of this conclusion consider a system whose coordinates are the generalized velocity u, the generalized displacement q and the generalized force F. The First Law is

$$\_E = dU - Fdq$$

where U and F are functions of u and q. Since the (u,q) surface is subdivided into a family of non-intersecting E-conservative curves σ(u,q) = constant where the constant can take on various values σ_1, σ_2, ..., and points on the surface may be determined by specifying the value of σ along with q, in all regions where the Jacobian of the transformation does not vanish, so that U, as well as F, may be regarded as functions of σ and q. Then

$$dU = \left[\frac{\partial U}{\partial \sigma}\right]_q d\sigma + \left[\frac{\partial U}{\partial q}\right]_\sigma dq$$

and

$$\_E = \left[\frac{\partial U}{\partial \sigma}\right]_q d\sigma + \left[\left[\frac{\partial U}{\partial q}\right]_\sigma - F\right] dq.$$

Since σ and q are independent variables this equation must be true for all values of dσ and dq.

Suppose dσ = 0 and dq ≠ 0. The provision that dσ = 0 is the provision for an E-conservative process in which dE = 0. Therefore, the coefficient of dq must vanish. Then, in order for σ and q to be independent and for dE to be zero when dσ is zero, the equation for dE must reduce to

$$\_E = \left[\frac{\partial U}{\partial \sigma}\right]_q d\sigma,$$

with

$$\left[\frac{\partial U}{\partial q}\right]_\sigma = F.$$

Defining a function λ by

$$\lambda \equiv \left[\frac{\partial U}{\partial \sigma}\right]_{q'}$$

then dE = λdσ where λ= λ(σ,q).

Now, in general, an infinitesimal of the type

Pdx + Qdy + Rdz + ...,

known as a linear differential form, or a Pfaffian expression, when it involves three or more independent variables, does not admit of an integrating factor. It is only because of the existence of the Second Law

Figure 3. Two reversible E-conservative curves, infinitesimally close. When the process is represented by a curve connecting the E-conservative curves, energy dE = λdσ is transferred.

that the differential form for dE referring to a physical system of any number of independent coordinates possesses an integrating factor.

Two infinitesimally neighboring reversible E-conservative curves are shown in Figure 3. One curve is characterized by a constant value of the function σ_A, and the other by a slightly different value σ_A + dσ = σ_B. In any process represented by a displacement along either of the two E-conservative curves dE = 0. When a reversible process connects the two E-conservative curves, energy $dE = \lambda d\sigma$ is transferred.

The various infinitesimal processes that may be chosen to connect the two neighboring reversible E-conservative curves, shown in Figure 3, involve the same change of σ but take place at different λ. In general λ is a function of u and q. However, it is obvious that λ may be expressed as a function of σ and u. To find the velocity dependence of λ consider two systems, one and two, such that in the first system there are two independent coordinates u and q and the E-conservative curves are specified by different values of the functions σ of u and q. When dE is transferred, σ changes by dσ and $dE = \lambda d\sigma$ where λ is a function of σ and q.

The second system has two independent coordinates u, and q' and the E-conservative curves are specified by different values of the function σ' of u and q'. When dE is transferred, σ' changes by dσ' and $dE = \lambda' d\sigma'$ where λ' is a function of σ' and u.

The two systems are related through the coordinate u in that both systems make up a composite system in which there are three independent coordinates u, q, and q' and the E-conservative curves are specified by different values of the function σ_c of these independent variables. To help visualize the situation it may be noted that the composite system is, in essence, two particles joined together and traveling with the same velocity but not sharing the same location.

Since $\sigma = \sigma(u,q)$ and $\sigma' = \sigma'(u,')$, using the equations for σ and σ', σ_c may be regarded as a function of u, σ and σ'.

For an infinitesimal process between two neighboring E-conservative surfaces specified by σ_c and $\sigma_c + d\sigma_c$, the energy transferred is $\pi E_c = \lambda_c d\sigma_c$ where λ_c is also a function of u, σ and σ'. Then

$$d\sigma_c = \left[\frac{\partial \sigma_c}{\partial u}\right]du + \left[\frac{\partial \sigma_c}{\partial \sigma}\right]d\sigma + \left[\frac{\partial \sigma_c}{\partial \sigma'}\right]d\sigma'. \quad (2.3)$$

Now suppose that in a process there is a transfer of energy dE_c between the composite system and an external reservoir with energies dE and dE' being transferred, respectively, to the first and second systems, then

$$\_E_c = \_E + \_E'$$

and

$$\lambda_c d\sigma_c = \lambda d\sigma + \lambda' d\sigma',$$

or

$$d\sigma_c = \left[\frac{\lambda}{\lambda_c}\right]d\sigma + \left[\frac{\lambda'}{\lambda_c}\right]d\sigma'. \quad (2.4)$$

Comparing Eqns. (2.3) and (2.4) for $d\sigma_c$ then

$$\frac{\partial \sigma_c}{\partial u} = 0.$$

Therefore σ_c does not depend on u, but only on σ and σ'. That is $\sigma_c = \sigma_c(\sigma,\sigma')$. Again comparing the two expressions for $d\sigma_c$ we find

$$\frac{\lambda}{\lambda_c} = \frac{\partial \sigma_c}{\partial \sigma} \quad also \quad \frac{\lambda'}{\lambda_c} = \frac{\partial \sigma_c}{\partial \sigma'}.$$

Therefore the two ratios λ/λ_c and $/\lambda_c$ are also independent of u, q and q'. These two ratios depend only on the σ's, but each separate λ must depend on the velocity as well (for example, if λ depended only on σ and on nothing else, the dE = λdσ would equal f(σ)dσ which is an exact differential). In order for each λ to depend on the velocity and at the same time for the ratios of the λ's to depend only on the σ's, the λ's must have the following structure:

$$\begin{aligned} \lambda &= \phi(u)\, f(\sigma), \\ \lambda' &= \phi(u)\, f'(\sigma'), \quad with \qquad (2.5) \\ \lambda_c &= \phi(u)\, g(\sigma,\sigma'). \end{aligned}$$

(The quantity λ cannot contain q, nor can contain q', since and $/\lambda_c$ must be functions of the σ's only.)

Referring now only to the first system as representative of any system of any number of independent coordinates, the transferred energy is, from Eqns. (2.5),

$$\_E = \phi(u)\, f(\sigma)\, d\sigma.$$

Since f(σ)dσ is an exact differential, the quantity 1/φ(u) is an integrating factor for dE. It is an extraordinary circumstance that, not only does an integrating factor exist for the dE of any system, but this integrating factor is a function of velocity only and is the same function for all systems. It would be nice if there were a simple way of deriving the functional form of φ(u). In thermodynamics we opted to take the easy way out by assuming that the integrating factor was simply the reciprocal of the temperature. However, for mechanical systems we will find the functional form of the integrating factor when we determine the equations of motion.

The fact that a system of two independent variables has a dE that always admits an integrating factor regardless of the axiom is interesting, but its importance in physics is not established until it is shown that the integrating factor is a function of velocity only and that it is the same function for all systems.

2.3 The Absolute Velocity and Einstein's Postulate.

The universal character of φ(u) makes it possible to define an absolute velocity. Consider a system of two independent variables q and u, for which two constant velocity curves and E-conservative curves are

Figure 4. Two constant velocity energy transfers, E_3 at u from b to c and E_3 at u_3 from a to d, between the same two conservative curves φ_1 and φ_2.

shown in Figure 4. Suppose there is a constant velocity transfer of energy E between the system and its surroundings at the velocity u, from a state b, on a E-conservative curve characterized by the value σ_1, to another state c, on another E-conservative curve specified by σ_2. Then since it is seen that

$$\_E = \phi(u)\int_{\sigma_1}^{\sigma_2} f(\sigma)\,d\sigma \quad \text{at constant } u.$$

For any constant velocity process between two other points a to d, at a velocity u_3 between the same E-conservative curves the energy transferred is

$$\_E(u_3) = \_E_3 = \phi(u_3)\int_{\sigma_1}^{\sigma_2} f(\sigma)\,d\sigma \quad \text{at constant } u_3.$$

Taking the ratio of

$$\frac{\Delta E}{\Delta E_3} = \frac{\varphi(u)}{\varphi(u_3)} = \frac{\text{a function of the vel. at which } \Delta E \text{ is transferred}}{\text{same function of vel. at which } \Delta E_3 \text{ is transferred}}.$$

Then the ratio of these two functions is defined by

$$\frac{\varphi(u)}{\varphi(u_3)} = \frac{\Delta E \text{ (between } \sigma_1 \text{ and } \sigma_2 \text{ at } u)}{\Delta E_3 \text{ (between } \sigma_1 \text{ and } \sigma_2 \text{ at } u_3)}$$

or

$$\_E = \left[\frac{\_E_3}{\phi(u_3)}\right]\phi(u).$$

By choosing some appropriate velocity u_3 it follows that the energy transferred at constant velocity between two given E-conservative curves decreases as φ(u) decreases, or the smaller the value of ΔE the lower the corresponding value of φ(u). When ΔE is zero φ(u) is also zero. The corresponding velocity u_0 such that $\varphi(u_0)$ is zero is the "absolute velocity". Therefore, if a system undergoes a constant velocity process between two E-conservative curves without an exchange of energy, the velocity at which this takes place is called the absolute velocity.

The definition of the absolute velocity requires constant velocity processes be considered. All Galilean frames of reference will display this process as one of constant velocity. Further, if all reference frames are to be of equal status then observers in all Galilean reference frames must share the dE = 0 constant velocity process equivalently. Furthermore, each observer will have the same value for the absolute velocity or else one of the frame will enjoy a privileged nature.

Just as the absolute temperature in classical thermodynamics is a limiting quantity we may suspect that the absolute velocity will also turn out to be a limiting quantity. Because of our experimental evidence that the speed of light behaves as a limiting velocity when electromagnetic forces are involved and the absolute velocity is independent of the force or type of system and is therefore unique, it must be the speed of light. Thus, the first two laws of the Dynamic Theory require Einstein's postulate concerning the speed of light.

To be more specific, the absolute velocity is unique for all Galilean frames of reference. There is one such velocity already known and that velocity
is the speed of light, c. Therefore, the absolute velocity must be the speed of light and the same for all Galilean observers. This is Einstein's postulate.

The above may be put on a more rigorous basis by observing that for the E-conservative process

$$\_E = 0 = \frac{\partial U}{\partial u^\alpha} du^\alpha + \left[\frac{\partial U}{\partial q^\alpha} - F_\alpha \right] dq^\alpha .$$

If dE = 0 is to be invariant for all points q^α then we must have

$$\frac{\partial U}{\partial q^\alpha} - F_\alpha = 0$$

and thus $du^\alpha = 0$, for all $\alpha = 1,2,3$. Thus the allowed transformations are those with constant velocities. This, of course, was just what was required by the statement of, or restriction to, a constant velocity process. Then all Galilean observers will agree upon the identification of an E-conservative system in absence of any work on the system.

Now let us suppose that at the time t, a system is at point $p(q^1, q^2, q^3)$ in Q. If our system is E-conservative and traveling at the absolute velocity, c, then in dt seconds it will be at the point $q^\alpha + dq^\alpha$ where $dq^\alpha = u^\alpha$ cdt. Now the speed is given by

$$\sqrt{u_\alpha u^\alpha} = \sqrt{g_{\alpha\beta} u^\alpha u^\beta} = c,$$

where $g_{\alpha ß}$ is the metric for the space and the metric is parameterized using the absolute velocity, c, which is the only velocity with an adequate definition thus far.

Now an observer in another frame Q' sees the system at the point P' given by (q^1, q^2, q^3) at the time t. In dt seconds the system will have moved to a point given by $q^\alpha + dq^\alpha$ and the speed will be given by

$$\sqrt{u'_\alpha u'^\alpha} = \sqrt{g'_{\alpha\beta} u'^\alpha u'^\beta} = c$$

or

$$g'_{\alpha\beta} d q'^\alpha d q'^\beta = c^2 dt^2$$

since the process must specify the E-conservative process at the absolute velocity, c.

But, since the Q observer must be Galilean then

$$q'^{\alpha} = a^{\alpha}_{\beta} q^{\beta} + a^{\alpha}_{4} t$$
$$t' = a^{4}_{\beta} q^{\beta} + a^{4}_{4}.$$

If we specialize so that $g_{\alpha\beta} = \delta_{\alpha\beta}$, $g'_{\alpha\beta} = \delta_{\alpha\beta}$, (ie. Euclidean) and we specify the relative motion between q and q' to be only in the q' direction, then our transformation is of the form

$$q' = a^{1}_{1} q' + a^{1}_{4} t$$
$$t' = a^{4}_{1} q' + a^{4}_{4} t. \tag{2.9}$$

Substituting equation (2.9) into (2.7) we find that

$$a^{1}_{4} a^{1}_{1} = c^{2} a^{4}_{1} a^{4}_{4}$$
$$(a^{1}_{1})^{2} = 1 + c^{2} (a^{4}_{1})^{2}$$
$$c^{2} (a^{4}_{4})^{2} = c^{2} + (a^{1}_{4})^{2}.$$

These are three equations in four unknowns. We need one further relation. But for the moment we have

$$(a^{1}_{4})^{2} = c^{2} (a^{4}_{4})^{2} - c^{2} = c^{2} [(a^{4}_{4})^{2} - 1]$$

and

$$(a^{1}_{1})^{2} = 1 + c^{2} (a^{4}_{1})^{2}.$$

If the q' moves with constant velocity v with respect to q then

$$\frac{dq'}{dt} = \left[\frac{a^{1}_{1} dq + a^{1}_{4} dt}{a^{4}_{1} dq + a^{4}_{4} dt}\right] = \left[\frac{a^{1}_{1} u + a^{1}_{4}}{a^{4}_{1} 7u + a^{4}_{4}}\right].$$

For u = 0,

$$\frac{dq'}{dt} = \frac{a^{1}_{4}}{a^{4}_{4}} = -v.$$

Thus $a_4{}^1 = -va_4{}^4$ which implies

$$(a^{1}_{4})^{2} = v^{2} (a^{4}_{4})^{2} = c^{2} [(a^{4}_{4})^{2} - 1],$$

or

$$a^{4}_{4} = \frac{\pm 1}{\sqrt{1 - \frac{v^{2}}{c^{2}}}} \equiv \pm \gamma.$$

Then $a_4{}^1 = \pm\gamma v$, and since

$$a^{1}_{1} = \sqrt{1 + c^{2} (a^{4}_{1})^{2}}$$

we have

$$a^{4}_{1} = \pm \left[\frac{\gamma v}{c^{2}}\right],$$

and

$$a^{1}_{1} = \pm \gamma.$$

We now have

$$q'^1 = \pm q^1 - v\gamma t$$
$$q'^2 = q^2$$
$$q'^3 = q^3$$
$$t' = \pm\left[\frac{\gamma v}{c^2}\right]u' + \gamma t.$$

Now if (dq'/dt) = v, this implies

$$\frac{dq'}{dt'} = \left[\frac{\pm v \pm v}{\pm\left(\frac{v^2}{c^2}\right) \pm 1}\right] = 0.$$

This means we must take the + sign for a_1^1. If (dq'/dt) = 0 we find

$$\frac{dq'}{dt'} = \left[\frac{\pm v}{\pm\left(\frac{v^2}{c^2}\right) \pm 1}\right] = -v$$

if we take the - sign for a_4^1. If (dq'/dt) = c then

$$\frac{dg'}{dt'} = \left[\frac{c - v}{\pm\left(\frac{cv^2}{c^2}\right) \pm 1}\right] = c$$

if the sign of a_1^4 is taken as - and the sign of a_4^4 is taken as +.

Thus, we have

$$a_1^1 = \gamma$$
$$a_4^1 = -\gamma v$$
$$a_1^4 = \left(\frac{-\gamma v}{c^2}\right)$$
$$a_4^4 = \gamma$$

or

$$
\begin{aligned}
q'^{1} &= \gamma\,(q^{1} - vt)\\
q'^{2} &= q^{2}\\
q'^{3} &= q^{3}\\
t' &= \gamma\left[t - \left(\frac{v}{c^{2}}\right)q'^{1}\right].
\end{aligned}
\tag{2.10}
$$

Equations (2.10) are the transformations of Einstein's Special Theory of Relativity, which, in Einstein's derivation needed only his postulate concerning the speed of light and the requirement that physics be the same for all Galilean observers. Here, in the Dynamic Theory we have shown that the Second Law requires Einstein's postulate and the transformations of Special Relativity for Galilean observers.

It should be noted that since the absolute velocity (or the speed of light) is unique the answer to whether there may be a different limiting velocity for different fundamental forces is answered by the Second Law. The Second Law states that there is only one limiting velocity independent of the type of force considered. Note that the function defined above as γ goes to zero as v tends to c. This is a property required of the integrating factor $\varphi(u)$ and raises suspicions concerning he functional form we will ultimately determine for φ.

2.4 The Concept of Entropy.

In a system of two independent variables, all states accessible from a given initial state by reversible E-conservative processes lie on a $\sigma(u,q)$ curve. The entire (u,q) space may be conceived as being filled by many non-intersecting curves of this kind, each corresponding to a different value of σ. In a reversible non-E-conservative process involving a transfer of energy dE, a system in a state represented by a point lying on a surface σ will change until its state point lies on another surface $\sigma + d\sigma$. Then dE $= \lambda d\sigma$, where $1/\lambda$, the integrating factor of dE, is given by $\lambda = \varphi(u)\ f(\sigma)$, and therefore $dE = \varphi(u)f(\sigma)d\sigma$ or

$$\frac{\bar{d}E}{\phi(u)} = f(\sigma)\,d\sigma .$$

Since σ is an actual function of u and q, the right-hand member is an exact differential, which may be denoted by dS; and

$$dS = \frac{\bar{d}E}{\phi(u)}$$

where S is the mechanical entropy of the system and the process is a reversible one.

The Dynamic Theory's Second Law may be used to prove the equivalent of Clausius's theorem, which is stated here without proof.

Theorem: In any cyclic transformation throughout which the velocity is defined, the following inequality holds:

$$\int \frac{E}{\phi(u)} \leq 0,$$

where the integral extends over one cycle of the transformation. The equality holds if the cyclic transformation is reversible. Then for an arbitrary transformation

$$\int_A^B \frac{E}{\phi(u)} \leq S(B) - S(A),$$

with the equality holding if the transformation is reversible. The proof of this statement may be seen by letting R and I denote respectively any reversible and any irreversible path joining A to B, as shown in Figure 5. For path R the assertion holds by definition of S. Now consider the cyclic transformation made up of I plus the reverse of R. From Clausius' theorem

$$\int_I \frac{E}{\phi} - \int_R \frac{E}{\phi} \leq 0,$$

or

$$\int_I \frac{\bar{d}E}{\phi} - \int_R \frac{\bar{d}E}{\phi} \equiv S(B) - S(A).$$

Another result of the Second Law is that the mechanical entropy of an isolated (dE = 0) system never decreases. This can be seen since an isolated system cannot exchange energy with the external world because dE = 0 for any transformation. Then by the previous property of the entropy,

$$S(B) - S(A) \leq 0$$

where the equality holds if the transformation is reversible. One consequence of the Second Law is that of all the possible transformations from one state A to another state B the one defined as the change in the entropy is the one for which the integral

$$I \equiv \int_A^B \frac{E}{\phi}$$

is a maximum. Thus

$$S(B) - S(A) = \max \int_A^B \left[\frac{1}{\phi} \frac{E}{d\tau} \right] d\tau,$$

where τ is a parameter that indicates position along the path from A to B, or

$$S(B) - S(A) = \max \int_A^B \left[\frac{1}{\phi}\left(\frac{dU}{d\tau}\right) - \frac{F}{\phi}\left(\frac{dq}{d\tau}\right) \right] d\tau .$$

If U = U(τ,q,u,du/dτ), then the change in the entropy is given by the integral

$$\_S = \int_A^B \left[\frac{1}{\phi}\left(\frac{dU}{d\tau}\right) - \frac{F}{\phi}\left(\frac{dq}{d\tau}\right) \right] d\tau .$$

The u and q which maximize ΔS will be denoted as v and x then, with U = U(x,v), F = F(x,v), and φ(v) the v and are given by the solution of the system of equations

$$\frac{d}{d\tau}\left[\frac{\partial G}{\partial x'}\right] - \frac{\partial G}{\partial x} = 0$$
$$\frac{d}{d\tau}\left[\frac{\partial G}{\partial v'}\right] - \frac{\partial G}{\partial v} = 0$$

where

$$G = \frac{1}{\phi}\left[\frac{\partial U}{\partial \tau} - F\frac{dx}{d\tau}\right],$$

x' = dx/dτ and v' = dv/dτ.

Thus, the Dynamic Theory's Second Law provides an answer to the question that is not contained within the scope of the First Law: In what direction does a process take place? The answer is that a process always takes place in such a direction as to cause an increase of the mechanical entropy in the universe. In the case of an isolated system, it is the entropy of the system that tends to increase. To find out, therefore, the equilibrium state of an isolated one-dimensional system, it is necessary merely to express the entropy as a function of q and u and to apply the usual rules of calculus to render the function a maximum. The equations, which describe the path the system takes toward the maximum of entropy, are the equations of motion for the isolated system. When the system is not isolated, there are other entropy changes to be taken into account.

2.5 Third Law.

The Second Law enables the mechanical entropy of a system to be defined up to an arbitrary additive constant. The definition depends on the existence of a reversible transformation connecting an arbitrarily chosen reference state 0 to the state under consideration. Such a reversible transformation always exists if both O and A lie on one sheet of the state surface. If two different systems are considered, the equation of the state surface may consist of several disjoint sheets. In such cases the kind of reversible path previously mentioned may not exist. Therefore, the Second Law does not uniquely determine the difference in entropy of two states A and B, if A defines a state of one system and B the state of another. For this determination a Third Law is needed. The Third Law may be stated, "The Mechanical Entropy of a system at the absolute velocity is a universal constant, which may be taken to zero." In the case of a purely thermodynamic system the absolute quantity is the absolute zero temperature, while for a mechanical system the absolute quantity is the absolute velocity. The Third Law implies that any energy capacity of a system must vanish at the absolute velocity. To see this, let R be any reversible path connecting a state of the system at the absolute velocity u_0 to the state A, whose entropy is to be found. Let C_R (u) be the energy capacity of the system along the path R. Then, by the Second Law,

$$S(A) = \int_{u_0}^{u_A} C_R(u)\left[\frac{du}{\phi(u)}\right].$$

But according to the Third Law, $S(A) \to 0$ as $u_A \to u_0$. Hence it follows that $C_R(u) \to 0$ as $u \to u_0$. In particular, C_R may be C_q or C_F.

The statement of the Third Law above reflects the restriction to mechanical work terms. A general statement of third law that is independent of the number or type of variables is "The generalized entropy of the system, when the integrating factor vanishes, is a universal constant, which may be taken to be zero."

B. General Relations

2.6 Energy and Maxwell's Relations.

In thermodynamics a discussion of equilibrium and stability conditions is best done if the enthalpy, Helmholtz's and Gibb's functions are defined first. Therefore, the mechanical analogues of these functions are defined here.

Each branch of physics such as thermodynamics and particle dynamics has its own developed procedures. If both branches can be described by the same basic laws, then the procedures developed in thermodynamics may prove to be useful in particle dynamics and vice versa. Once the mechanical enthalpy, mechanical Helmholtz's and mechanical Gibbs' functions are defined, it is then easy to write down the resulting mechanical Maxwell and mechanical energy capacity relations.

To begin the development of the Maxwell relations, the mechanical entropy was defined as

$$dS \equiv \frac{E}{\phi(u)}.$$

Then, since dE = dU -Fdq,

$$dS = \frac{dU}{\phi} - \frac{F}{\phi}dq,$$

where

$$dU = \phi(u)dS + Fdq$$

(2.11)

Define the mechanical enthalpy as H = U - Fq. Then

$$dH = \phi(u)dS - qdF.$$

(2.12)

Therefore

$$\left[\frac{\partial H}{\partial S}\right]_F = \phi(u) \quad ; \quad \left[\frac{\partial H}{\partial F}\right]_S = -q.$$

The mechanical Helmholtz's function can be defined as K = U - φ(u)S, and

$$dK = dU - \frac{d[\phi(u)]}{du}Sdu - \phi(u)dS$$

or, with φ'(u) = dφ/du,

$$dK = -S\phi'(u)du - Fdq.$$

(2.13)

This leads to

$$\left[\frac{\partial K}{\partial u}\right]_q = -S\phi'(u) \quad ; \quad \left[\frac{\partial K}{\partial q}\right]_u = \phi(u)F.$$

The mechanical Gibb's function may be defined as G = H - φ(u)S then

$$dG = -\phi'(u)Sdu + qdF,$$

(2.14)

so that

$$\left[\frac{\partial G}{\partial u}\right]_F = -\phi'(u)S \;\; ; \;\; \left[\frac{\partial G}{\partial F}\right]_u = q.$$

From the differential Eqns. (2.11), (2.12), (2.13), and (2.14) the Maxwell relations for a mechanical system may be written:

$$\phi'(u)\left[\frac{\partial u}{\partial q}\right]_S = \left[\frac{\partial F}{\partial S}\right]_q$$
$$\phi'(u)\left[\frac{\partial u}{\partial F}\right]_q = -\left[\frac{\partial q}{\partial S}\right]_F$$
$$\phi'(u)\left[\frac{\partial S}{\partial q}\right]_u = -\left[\frac{\partial F}{\partial u}\right]_q$$
$$\phi'(u)\left[\frac{\partial S}{\partial F}\right]_u = \left[\frac{\partial q}{\partial u}\right]_F .$$

(2.15)

The energy capacity at the position q can be defined as

$$C_q \equiv \left[\frac{\partial E}{\partial u}\right]_q = \phi(u)\left[\frac{\partial S}{\partial u}\right]_q .$$

Define the energy capacity with a constant force as

$$C_F \equiv \left[\frac{\partial E}{\partial u}\right]_F = \phi(u)\left[\frac{\partial S}{\partial u}\right]_F$$

then

$$(C_q - C_F) = \frac{\phi(u)}{\phi'(u)}\left[\frac{\partial q}{\partial u}\right]_F\left[\frac{\partial F}{\partial u}\right]_q ,$$

and

$$\frac{C_F}{C_q} = \frac{\left[\frac{\partial F}{\partial q}\right]_S}{\left[\frac{\partial F}{\partial q}\right]_u} .$$

The three generalized laws have been formulated and a few results of these laws have been seen. The next step is to derive the stability conditions to obtain the quadratic forms necessary for a metric. The derivation of the equilibrium and stability conditions is identical to the derivation of the thermodynamic equilibrium and stability conditions with the variables changed to represent the mechanical variables q, u, S and F instead of the thermodynamic variables T, V, S and P.

2.7 Equilibrium Conditions.

To establish the criteria for equilibrium, consider, Clausius's theorem

$$\int_A^B \frac{E}{\phi}_I - \int_A^B \frac{E}{\phi}_R \leq 0,$$

or

$$\int_A^B \frac{E}{\phi}_I \leq \int_A^B \frac{E}{\phi}_R \equiv S(B) - S(A).$$

For an E-conservative system dE = 0, then $\Delta S \geq 0$, or $S(B) \geq S(A)$. Therefore the mechanical entropy tends toward a maximum so that spontaneous changes in an E-conservative system will always be in the direction of increasing mechanical entropy.

Now by First Law $\Delta E = \Delta U - F\Delta q$. Therefore \$\varphi\Delta S \geq \Delta U - F\Delta q$, which is analogous to the Clausius inequality in thermodynamics. Now consider a virtual displacement $(U,q) \rightarrow (U + \delta U, q + \delta q)$, which implies a variation $S \rightarrow S + \delta S$ away from equilibrium. The restoration of equilibrium from the varied state $(U + \delta U, q + \delta q) \rightarrow f(U,q)$ will then certainly be a spontaneous process, and by the Clausius inequality $\varphi(-\delta S) > -(\delta U - F\delta q)$. Hence, for variations away from equilibrium, the general inequality

$$\delta U - F\delta q - \phi\delta S > 0 \tag{2.16}$$

must hold. The inequality sign is reversed from the sign in Clausius' inequality because hypothetical variations δ away from equilibrium are considered rather than real changes toward equilibrium.

In a spontaneous process,

$\varphi\Delta S \geq \Delta E_{rev} = \Delta U$ + work done by the system.

The "work" consists of two parts. One part is the work done by the negative of the force F. It may be positive or negative, but it is <u>inevitable</u>. Only the rest is free energy, which is available for some useful work. This latter part may be written as

$$\Delta A = \Delta E_{rev} - \Delta U + F\Delta q.$$

The maximum of A is

$$\Delta A_{max} = \phi\Delta S - \Delta U + F\Delta q, \tag{2.17}$$

which is obtained when the process is conducted reversibly.

The least work, δA_{min}, required for a displacement from equilibrium must be exactly equal to the maximum work in the converse process whereby the system proceeds spontaneously from the 'displaced' state to equilibrium (otherwise a perpetual motion machine may be constructed. Corresponding to Eqn. (2.17) then,

$$\delta A_{min} = \delta U - F\delta q - \phi\delta S.$$

The equilibrium criteria may then be expressed as $\delta A_{min} \geq 0$. In words: At equilibrium the mechanical free energy is a minimum. Any displacement from this state required work.

2.8 Stability Conditions.

To decide whether or not an equilibrium is stable, the inequality sign in Eqn. (2.16) must be ensured. The conditions for stability may take different forms depending upon which variables are taken as the independent variables.

To derive the stability conditions when q and S are taken as the independent variables consider the terms of second order in small displacements beginning with the general condition

$$\delta U - F\delta q - \phi\delta S > 0.$$

Choose U = U(q,S), which, because of the identity

$$\phi dS = dU - Fdq$$

is a natural choice for the independent variables, and expand δU in powers of the δq and δS

$$\begin{aligned} \delta U &= \phi\delta S + F\delta q \\ &+ \frac{1}{2}\left[\frac{\partial^2 U}{\partial q^2}\right]\delta q^2 + 2\left[\frac{\partial^2 U}{\partial q\partial S}\right]\delta q\delta S + \left[\frac{\partial^2 U}{\partial S^2}\right]\delta S^2 \\ &+ \text{terms of third order...} \end{aligned} \tag{2.18}$$

The inequality (2.16) then shows that in Eqn. (2.18),

second order terms + third order terms + . . . > 0.

Retaining only the second order terms, the criterion of stability is that a quadratic differential form be positive definite;

$$\frac{\partial^2 U}{\partial q^2}\delta q^2 + 2\frac{\partial^2 U}{\partial q \partial S}\delta q \delta S + \frac{\partial^2 U}{\partial S^2}\delta S^2 > 0. \tag{2.19}$$

If this is to hold true for arbitrary variations in δq and δS, the coefficients must satisfy the following:

$$\frac{\partial^2 U}{\partial q^2} > 0 \;\; ; \;\; \left[\frac{\partial^2 U}{\partial S^2}\right]\left[\frac{\partial^2 U}{\partial q^2}\right] - \left[\frac{\partial^2 U}{\partial q \partial S}\right] > 0.$$

An alternate approach is seen when u and q are considered to be the independent variables, a quadratic form in δu and δq may be found by using K = U - φS so that

$$\delta K = \delta U - \phi \delta S - \frac{d\phi}{du} S \delta u -$$

The terms δSδu cannot be neglected because in Clausius's inequality, which is the actual stability condition, the variations are finite, and therefore, from Eqn. (2.16) the following is obtained:

Expanding in powers of δu and δq,

$$\delta K = F\delta q - \frac{d\phi}{du} S\delta u + \frac{1}{2}\frac{\partial^2 K}{\partial q^2}\delta q^2 + \frac{\partial^2 K}{\partial q \partial u}\delta q \delta u + \frac{1}{2}\frac{\partial^2 K}{\partial q^2}\delta q^2 + \ldots$$

and

$$\delta S \delta u = \frac{1}{\phi}\frac{\partial U}{\partial u}\delta u^2 + \frac{1}{\phi}\left[\frac{\partial U}{\partial q} - F\right]\delta q \delta u.$$

But

$$\frac{\partial K}{\partial u} = \frac{\partial U}{\partial u} -$$

and

$$\frac{\partial K}{\partial q} = F.$$

Therefore

$$\frac{\partial^2 K}{\partial u \partial q} = \frac{\partial F}{\partial u} = -\frac{\partial \phi}{\partial u}\left[\frac{1}{\phi}\right]\left[\frac{\partial U}{\partial q} - F\right]\delta q \delta u.$$

and

$$\frac{\partial^2 K}{\partial u^2} = -\frac{\partial^2 \phi}{\partial u^2} S - \frac{d\phi}{du}\left[\frac{1}{\phi}\right]\frac{\partial U}{\partial u}.$$

Then

$$\left[\frac{d\phi}{du}\right]\delta S \delta u = -\left[\frac{\partial^2 \phi}{\partial u^2} S + \frac{\partial^2 K}{\partial u^2}\right](\delta u)^2 - \frac{\partial^2 K}{\partial u \partial q}\delta u \delta q$$

and the quadratic form in δu and δq is

$$\frac{\partial^2 K}{\partial q^2}(\delta q)^2 - \left[\frac{\partial^2 K}{\partial u^2} + 2\frac{\partial^2 \phi}{\partial u^2} S\right](\delta u)^2 > 0.$$

Since

$$\left[\frac{\partial K}{\partial q}\right]_u = F,$$

then

$$\frac{\partial^2 K}{\partial q^2} = \left[\frac{\partial F}{\partial q}\right]_u > 0.$$

Other quadratic forms may be derived by using different independent variables; however, these two quadratic forms will suffice for this development.

C. Geometry

2.9 Geometry Required by the Fundamental Laws.

There is nothing that specifies which of the quadratic forms coming from the stability conditions should be adopted as the metric. Thus the choice may be based upon simplicity and/or applicability. However, it becomes obvious that if we choose one of the forms using the velocity as our metric and then obtain equations of motion, then the equations of motion will become third order differential equations in time since the velocity is itself first order and the equations of motion are second order differential equations.

The fact that these equations of motion will become third order differential equations in time displays a time asymmetry that appears to correspond to nature. However, third order equations are difficult or impossible to solve.

To avoid the difficulty of third order equations of motion, suppose we adopt the quadratic form of Eqn. (2.19) as the metric for our system. Thus we are adopting a manifold with coordinates of space and mechanical entropy. This choice is not totally arbitrary because we wish to choose a metric that will display the metric of Einstein's Special and General Relativity as subsets of our metric. Looking toward this objective guides us in the choice of metric.

It now becomes desirable to extend our system beyond the dimensionality used thus far. Such an extension brings up a question concerning the integrating factor. With one work term the differential of the entropy was written as

$$dS = \frac{\_E}{\phi} = f_i \, d\sigma_i .$$

Then if for each dimension the exchange of energy is denoted by dE_j, then

$$dS_i = \frac{\_E_i}{\phi_i} = f_i \, d\sigma_i ,$$

where there is no summation intended for $f_i \, d\sigma_i$. Since each dS_i is a perfect differential, then the total change in mechanical entropy may be written as

$$dS = \sum_i dS_i = \sum_i \frac{\_E_i}{\phi_i} = \sum_i f_i \, d\sigma_i .$$

However, the question which arises is whether there exists a single integrating factor φ such that

$$dS = \frac{\_E}{\phi} = \sum_i \frac{\_E_i}{\phi_i} = \sum_i f_i \, d\sigma_i .$$

To see this consider the element of work considered before as

$$\_W = \sum_i F_i \, dq^i ; \quad i = 1,\ldots,n.$$

Since each dU_i is in itself a perfect differential, then $dU = \sum_i dU_i$ so that

$$\_E = \sum_i dU_i - \sum_i F_i \, dq^i = \sum_i (dU_i - F_i \, dq^i)$$

or

$$\_E = \sum_i \_E_i .$$

If the system is total E-conservative in the sense that

$$\_E = \sum_i \_E_i = 0,$$

then dE = 0 is a Pfaffian differential equation. This equation is integrable and has an integrating factor φ. The integrability is guaranteed by the Second Law since it is impossible to go from one initial state to <u>any</u> neighboring state. Then, just as in the one-dimensional case, the perfect differential follows:

$$dS = \frac{\_E}{\phi} = \sum_i \frac{\_E_i}{\phi}.$$

But since

$$\_E = \sum_i \phi_i f_i d\sigma_i,$$

then

$$dS = \sum_i \phi_i \frac{f_i}{\phi} d\sigma_i.$$

Now following the same argument presented in Section 2.2 concerning the composite system, $dE = \lambda d\sigma$ where σ is a function of all the σ_i and the u_i. Therefore, since $dE_i = \lambda_i d\sigma_i$, then

$$\_E = \sum_i \lambda_i \left[\frac{\partial \phi_i}{\partial \sigma} d\sigma + d\sigma_i\right].$$

Now

$$d\sigma = \sum_i \left[\frac{\partial \phi}{\partial u^i} du^i + \frac{\partial \lambda}{\partial \sigma_i} d\sigma_i\right]$$

so that $dE = \sum_i dE_i$ or $\lambda d\sigma = \sum_i \lambda_i d\sigma_i$ and

$$d\sigma = \sum_i \left[\frac{\lambda_i}{\lambda}\right] d\sigma_i.$$

It follows that the $\partial\sigma/\partial u^i = 0$ and that the ratios λ_i/λ are also independent of the q^i. Therefore the λ's have the form $\lambda_i = \varphi\ f_i$ and $\lambda = \varphi\ F(\sigma_i, \sigma_i, \ldots, \sigma_n)$ and also

$$\_S = f d\sigma = \sum_i F\left[\frac{\lambda_i}{\lambda}\right] d\sigma_i = \sum_i \left[\frac{\lambda_i}{\phi} d\sigma_i\right] = \sum_i f_i d\sigma_i.$$

The right hand side is a perfect differential and therefore so is the left.

Since each λ_i/f_i is an integrating factor and λ/F is also an integrating factor, it follows that $\varphi(u^1, u^2, \ldots, u^n)$ is an integrating factor for the dE as well as for $dE_i = \sum_i dE_i$. Therefore

$$dS = \frac{E}{\phi} = \sum_i \frac{E_i}{\phi}.$$

The importance of this question may be seen in terms of the difficulty that would be created if a universal integrating factor could not be found. For then each additional work term would require its own integrating factor to be determined individually.

Thus assured that an overall integrating factor exists, then the existence of an overall entropy function is guaranteed so that

$$dS = \frac{E}{\phi} = \frac{dU}{\phi} - \frac{F_i}{\phi} dq^1$$

for any i and the quadratic form may be extended to include three spatial work terms and thus becomes

$$\frac{\partial^2 U}{\partial S^2}(dS)^2 + \frac{\partial^2 U}{\partial S \partial q^\alpha}(dS)(dq^\alpha) + \frac{\partial^2 U}{\partial q^\alpha \partial q^\beta}(dq^\alpha)(dq^\beta) > 0;$$
$$\alpha, \beta = 1,2,3.$$

Adopting this quadratic form as the metric of a general system whose thermodynamic variables are held fixed, we may then write this metric as

$$(dS)^2 = h_{ij} dq^i dq^j; \quad i, j = 0,1,2,3, \tag{2.20}$$

where the summation convention is used and

$$h_{ij} = \frac{\partial^2 U}{\partial q^i \partial q^j},$$

with $q^0 = S/F_0$, the scaled mechanical entropy for dimensional correctness.

Thus, the stability conditions provide a metric in the four-dimensional manifold of space-mechanical entropy. However, the existing relativistic theories are theories in a space-time manifold. Therefore, if these theories are to be contained within the Dynamic Theory, then the space-time manifold must be found within the Dynamic Theory.

The arc length s in the space-mechanical entropy manifold may be parameterized by choosing $ds = u_0 dt = cdt$, where $u_0 = c$ is the unique velocity appearing in the integrating factor of the second postulate. There are two reasons for choosing the unique velocity. First, it is the only well-defined velocity we have thus far. Secondly, we may look ahead to the metric of the Special Theory of Relativity. The metric may now be written as

$$c^2 (dt)^2 = h_{ij}\, dq^i\, dq^j \;;\; i, j = 0,1,2,3.$$

(2.21)

Now suppose the systems considered are restricted to only E-conservative systems. Then the principle of increasing mechanical entropy may be imposed in the form of the variational principle

$$\delta \int \sqrt{(dq^0)^2} = 0.$$

In order to use this variation principle, Eqn. (2.21) may be expanded, solved for (dq^0) and squared to arrive at the quadratic form

$$(dq^0)^2 = \frac{1}{h_{00}}\left[c^2 (dt)^2 + 2\, h_{0\alpha}\, A dt dq^\alpha - h_{\alpha\beta}\, dq^\alpha\, dq^\beta\right]$$

(2.22)

where

$$A = \frac{h_{0\alpha}}{h_{00}} u^\alpha +_- \sqrt{\frac{c^2}{h_{00}} + \frac{h_{\alpha\beta}}{h_{00}} u^\alpha u^\beta + \frac{h_{0\alpha}}{h_{00}} (u^\alpha)^2}$$

with $u^\alpha = dq^\alpha / dt$.

By defining $x^0 = ct$, $x^\alpha = q^\alpha$; $\alpha = 1, 2, 3$, then Eqn. (2.22) may be written as

$$(dq^0)^2 = \frac{1}{f} g'_{ij}\, dx^i\, dx^j \;;\; i, j = 0,1,2,3$$

(2.23)

where $f = h_{00}$. This metric obviously reduces, in the Euclidean limit of constant coefficients, to the metric of Minkowski's space-time manifold of Special Relativity. It is interesting to note that in the metric of Eqn. (2.22) the difference in the sign on the time and space elements of the metric come from stability conditions given in terms of space and mechanical entropy while the variational principle was taken to be the Entropy Principle. In this fashion the Second Law guarantees the limiting aspect found in Einstein's Special Theory of Relativity.

In his General Theory of Relativity, Einstein assumed the space-time manifold to be Riemannian. However, this assumption involves the a priori assumption that the scalar product be invariant. This assumption was later questioned by Weyl in his generalization of geometry. From the viewpoint that the adopted postulates of the Dynamic Theory should contain the other theories it then becomes desirable to determine whether or not these postulates specify the geometry of the $(dq^0)^2$ space-time manifold. More particularly do the adopted postulates lead to a geometry that includes the geometry of current theories? To arrive at a more general geometry would not be a limitation for it would certainly include the others.

Recalling Eqn. (2.23), we can define

$$(dq^0)^2 = \frac{1}{f} g'_{ij} dx^i dx^j \equiv \frac{1}{f}(d\sigma)^2 \equiv g_{ij} dx^i dx^j . \tag{2.24}$$

Now the Second Law guarantees the existence of the function mechanical entropy and that dq^0 be a perfect differential; therefore

$$dq^0 = q^0{}_i dx^i , \tag{2.25}$$

where $q^0{}_i = \partial q^0/\partial x^i$. Then the exactness of dq^0 is stated by

$$q^0{}_{i|j} - q^0{}_{j|i} = 0. \tag{2.26}$$

By defining the parallel displacement of a vector to be

$$d\zeta_i = \Gamma^v{}_{is} dx^s \zeta_v \tag{2.27}$$

and using Eqns. (2.26) and (2.27) it may be seen that the connections must be symmetrical, or

$$\Gamma^v{}_{ik} = \Gamma^v{}_{ki} . \tag{2.28}$$

This result should not be taken to mean that only symmetric connections need to be considered. Rather it means that given the $_{ij}$'s that maximize $(dq^0)^2$, then the connections are symmetrical. However, since a variational principle must be used to determine the $_{ij}$'s, then both symmetric and antisymmetrical connections will have to be considered.

In Weyl's generalization of geometry he found it necessary to assume the symmetry of the connections. He proved a theorem showing that the symmetry of the connections guaranteed the existence of a local

Euclidean limiting manifold and used this theorem in support of the symmetry assumption. Here we find that the Second Law requires that the connections formed by the solution coefficients must be symmetrical thus guaranteeing, through Weyl's theorem, the existence of a local Euclidean geometry within the Dynamic Theory.

Suppose now we consider whether the order of differentiating the change in entropy makes any difference. This means that we must use symmetric connections since the actual change in entropy will be determined by the metric coefficients that generate a maximum. Therefore, consider the difference

$$\_(dq^0)^2 = \frac{\partial^2 (dq^0)^2}{\partial x^i \partial x^j} -$$

Since $(dq^0)^2 = q^0{}_i q^0{}_j dx^i dx^j$ from Eqn. (2.25), using Eqn. (2.24) we find $q^0{}_i q^0{}_j = g_{ij}$. Then

$$\frac{\partial (dq^0)^2}{\partial x^k} = [q^0{}_{j|k} q^0{}_i + q^0{}_{i|k} q^0{}_j]\, dx^i dx^j + (d\, q^0{}_k)^2 .$$

Thus

$$\frac{\partial^2 (dq^0)^2}{\partial x^k \partial x^l} = [q^0{}_{j|k|l} q^0{}_i + q^0{}_{j|k} q^0{}_{i|l} + q^0{}_{i|k|l} q^0{}_j + q^0{}_{i|k} q^0{}_{j|l}]\, dx^i dx^j$$
$$+ 2\, q^0{}_{l|k} q^0{}_l + 2\, q^0{}_{k|l} q^0{}_k .$$

Likewise

$$\frac{\partial^2 (dq^0)^2}{\partial x^l \partial x^k} = [q^0{}_{j|l|k} q^0{}_i + q^0{}_{j|l} q^0{}_{i|k} + q^0{}_{i|l|k} q^0{}_j + q^0{}_{i|l} q^0{}_{j|k}]\, dx^i dx^j$$
$$+ 2\, q^0{}_{k|l} q^0{}_k + q^0{}_{l|k} q^0{}_l .$$

Therefore the difference must be

$$\_(dq^0)^2 = [(q^0{}_{j|k|l} - q^0{}_{j|l|k}) q^0{}_i + (q^0{}_{i|k|l} - q^0{}_{i|l|k}) q^0{}_j]\, dx^i dx^j .$$

Using the definition Eqn. (2.27) we see that

$$d\, q^0{}_i = \Gamma^r{}_{is} dx^s q^0{}_r ,$$
$$q^0{}_{i|k} = q^0{}_r \Gamma^r{}_{ik} , also$$
$$q^0{}_{k|i} = q^0{}_r \Gamma^r{}_{ki} .$$

Now

$$
\begin{aligned}
q^0{}_{i|k|l} &= \frac{\partial}{\partial x^l}[\,q^0{}_r \Gamma^r{}_{ik}\,] \\
&= q^0{}_{r|l}\Gamma^r{}_{ik} + q^0{}_r \frac{\partial \Gamma^r{}_{ik}}{\partial x^l} \\
&= q^0{}_s \Gamma^s{}_{rl}\Gamma^r{}_{ik} + q^0{}_r \frac{\partial \Gamma^r{}_{ik}}{\partial x^l} \\
&= q^0{}_r \left[\Gamma^r{}_{sk}\Gamma^s{}_{ik} + \frac{\partial \Gamma^r{}_{ik}}{\partial x^l} \right].
\end{aligned}
$$

Similarly

$$
q^0{}_{i|l|k} = q^0{}_r \left[\Gamma^r{}_{sk}\Gamma^s{}_{il} + \frac{\partial \Gamma^r{}_{il}}{\partial x^k} \right].
$$

Therefore

$$
q^0{}_j\,[\,q^0{}_{i|k|l} - q^0{}_{i|l|k}\,] = q^0{}_i\, q^0{}_r \left[\frac{\partial \Gamma^r{}_{ik}}{\partial x^l} - \frac{\partial \Gamma^r{}_{il}}{\partial x^k} + \Gamma^r{}_{sl}\Gamma^s{}_{ik} - \Gamma^r{}_{sk}\Gamma^s{}_{il} \right].
$$

Then defining the vector curvature as

$$
R^r{}_{ilk} \equiv \frac{\partial \Gamma^r{}_{ik}}{\partial x^l} -
$$

the difference may be written as

$$
\_(dq^0)^2 = [\,q^0{}_j\, q^0{}_r R^r{}_{ilk} + q^0{}_i\, q^0{}_r R^r{}_{jlk}\,]\, dx^i dx^j .
$$

However, recall that $q^0{}_i q^0{}_j = {}_{ij}$; then

$$
\_(dq^0)^2 = [\,g_{jr} R^r{}_{ilk} + g_{ir} R^r{}_{jlk}\,]\, dx^i dx^j .
$$

But $g_{ri} = g_{ir}$ and $R_{ijkl} = g_{ir}R^r{}_{jkl}$, so that

$$
\_(dq^0)^2 = [\,R_{jilk} + R_{ijlk}\,]\, dx^i dx^j .
$$

So the difference will vanish if $R_{jilk} = -R_{ijlk}$. Now since

$$
(dq^0)^2 = q^0{}_i\, q^0{}_j\, dx^i dx^j = g_{ij}\, dx^i dx^j ,
$$

differentiation will result in

$$
d(dq^0)^2 = d(\,q^0{}_i\, q^0{}_j\, dx^i dx^j\,) = d(\,g_{ij}\, dx^i dx^j\,)
$$

or

$$d\, q^0{}_i q^0{}_j dx^i dx^j + q^0{}_i dq^0{}_j dx^i dx^j + q^0{}_i q^0{}_j d(dx^i dx^j) = dg_{ij} dx^i dx^j + g_{ij} d(dx^i dx^j),$$

which can be written as

$$\Gamma^r{}_{is} dx^s q^0{}_r q^0{}_j dx^i dx^j + q^0{}_i \Gamma^r{}_{js} dx^s q^0{}_r dx^i dx^j + q^0{}_i q^0{}_j d(dx^i dx^j) = dg_{ij} dx^i dx^j + g_{ij} d(dx^i dx^j).$$

But $g_{ij} = q^0{}_i q^0{}_j$. Therefore

$$\Gamma^r{}_{is} dx^s g_{rj} + \Gamma^r{}_{js} dx^s g_{ir} = dg_{ij}$$

or

$$g_{rj} \Gamma^r{}_{is} + g_{ri} \Gamma^r{}_{js} = \frac{\partial g_{ij}}{\partial x^s}$$

and

$$\Gamma_{jis} + \Gamma_{ijs} = \frac{\partial g_{ij}}{\partial x^s}. \tag{2.30}$$

Now interchange jis to sij to get

$$\Gamma_{sij} + \Gamma_{jsi} = \frac{\partial g_{is}}{\partial x^j}. \tag{2.31}$$

Then interchange jis to isj so that

$$\Gamma_{isj} + \Gamma_{sij} = \frac{\partial g_{si}}{\partial x^j}. \tag{2.32}$$

Add Eqns. (2.31) and (2.32) and subtract Eqn. (2.30).

$$\Gamma_{sij} + \Gamma_{jsi} + \Gamma_{isj} + \Gamma_{sij} - \Gamma_{jis} -$$

or

$$\Gamma_{sij} = \frac{1}{2}\left[\frac{\partial g_{si}}{\partial x^j} + \frac{\partial g_{sj}}{\partial x^i} + \frac{\partial g_{ij}}{\partial x^s}\right]$$

(2.33)

and

$$\Gamma^r{}_{ij} = g^{rs}\Gamma_{sij} = \frac{g^{rs}}{2}\left[\frac{\partial g_{si}}{\partial x^j} + \frac{\partial g_{sj}}{\partial x^i} + \frac{\partial g_{ij}}{\partial x^s}\right].$$

Now by using the symmetry of g_{ij} it can be shown that

$$R_{jilk} = - R_{ijlk}$$

and therefore $\Delta(dq^0)^2 = 0$. This is the necessary and sufficient condition that the differential entropy change may be transferred from an initial point to all points of the space in a manner that is independent of the path.

The distinguishing feature of Riemannian geometry is the invariance of the scalar product under a vector transplantation. Therefore to determine whether the $(dq^0)^2$ space is a Riemannian space, consider the vector $_i$ and $_i$. Now since $\xi_i = g_{ij}\xi^j$ and

$$d\xi_i = \Gamma^r{}_{is}dx^s\xi_r = \Gamma^r{}_{is}dx^s g_{rk}\xi^k = \frac{\partial g_{ij}}{\partial x^s}\xi^j dx^s + g_{ij}d\xi^j,$$

then

$$g_{ij}d\xi^j = \Gamma^r{}_{is}dx^s g_{rk}\xi^k - \frac{\partial g_{ij}}{\partial x^s}\xi^j dx^s.$$

Or, since $g^{ij}g_{ij} = \delta^i_i = 1$ and

$$\frac{\partial g_{ij}}{\partial x^s} = \Gamma_{jis} + \Gamma_{ijs},$$

then

Thus the change in the covariant and the contravariant vectors are given by

Now consider the change in the scalar product $\xi_i\eta^i$. Then

$$d(\xi_i \eta^i) = d\xi_i \eta^i + \xi^i d\eta_i$$
$$= \Gamma^r{}_{is} dx^s \xi_r \eta^i + \xi_i(-\Gamma^i{}_{rs} dx^s \eta^r)$$
$$= \Gamma^r{}_{is} dx^s \xi_r \eta^i - \Gamma^i{}_{rs} dx^s \xi_i \eta^r .$$

Renaming the indices in the second term yields

$$d(\xi_i \eta^i) = (\Gamma^r{}_{is} \xi_r \eta^i - \Gamma^r{}_{is} \xi_r \eta^i) dx^s .$$

Thus the geometry of the $(dq^0)^2$ manifold is Riemannian.

Next consider the question of what is the geometry of the $(d\sigma)^2$ space? Equation (2.24) shows that we may write $(d\sigma)^2 = f(dq^0)^2$, which is reminiscent of Weyl's generalized geometry. Further we have

$$g'_{ij} = f\, g_{ij} .$$

Then in the sigma space an arbitrary vector ξ^i would have a length given by the self-scalar product

$$l' = ||\xi||^2 = g'_{ij} \xi^i \xi^j - \tag{2.34}$$

where l is the length of the vector in the entropy space.

If we differentiate Eqn. (2.34), we have

$$2l'd'l = l'^2 \frac{\partial f}{\partial x^i} dx^i + 2fldl.$$

However, in the entropy space the length of the vector is unchanged under parallel displacement so that

$$dl' = \frac{1}{2} \frac{\partial f}{\partial x^i} dx^i \frac{l'}{f} = \frac{1}{2} \frac{\partial lnf}{\partial x^i} dx^i l'. \tag{2.35}$$

Comparing Eqn. (2.35) with the definition of the parallel displacement of a vector, Eqn. (2.27), we find that

$$\phi_i = \frac{\partial lnf}{\partial x^i}$$

plays a role similar to that of the connections $\Gamma^i{}_{jk}$ in the definition of parallel displacement of a vector. Therefore we shall define the change in the length of a vector under displacement to be

$$dl = (\phi_i dx^i)l.$$

(2.36)

This is the same definition Weyl made in his generalization of geometry. However, there is a difference in the way it was obtained. Weyl chose this definition in analogy with the connections Γ and the definition then led to the second more general metric. In this theory the fundamental laws lead us to two metrics and Eqn. (2.35) for the change in the length of a vector under displacement. Therefore, we have no choice.

Thus within the Dynamic Theory Eqn. (2.35) is a derived equation and Eqn. (2.36) only renames the logarithmic derivative.

Using Eqn. (2.36) we may obtain, in general,

$$dl^2 = 2l^2(\phi_i dx^i) = d(g_{ij}\xi^i\xi^j)$$
$$= g_{ij|k}\xi^i\xi^j dx^k + g_{ij}\Gamma^i{}_{lk}\xi^l\xi^j dx^k + g_{ij}\Gamma^j{}_{lk}\xi^i\xi^l dx^k.$$

Renaming the various summation indices, rearranging terms, and using the length of a vector, we obtain

$$[g_{ij|k} + g_{lj}\Gamma^l{}_{ik} + g_{il}\Gamma^l{}_{jk}]\xi^i\xi^j dx^k = 2g_{ij}\phi_k\xi^i\xi^j dx^k.$$

Since this must hold for arbitrary choice of ξ^i and dx^k, we conclude that

$$(g_{ij|k} - 2g_{ij}\phi_k) + g_{lj}\Gamma^l{}_{ik} + g_{li}\Gamma^l{}_{jk} = 0.$$

This is the same system of linear equations for the connections $\Gamma^i{}_{jk}$ as Eqn. (2.30) except that the inhomogeneous term $_{ijk}$ has now to be replaced by g_{ijk} - $2g_{ij}\,\varphi_k$. Therefore the same linear algebra as before leads to

$$\Gamma^i{}_{jk} = -\begin{pmatrix} i \\ jk \end{pmatrix} + g^{li}[g_{lj}\phi_k + g_{lk}\phi_j -$$

(2.37)

where $(^i{}_{jk})$ is the usual Christoffel symbol of the second kind.

Now, since the entropy space is Riemannian, then in the entropy space we have $\varphi'_i=0$ and $\Gamma'^i{}_{jk} = -(^i{}_{jk})$ and the length l of a vector is unchanged under parallel displacement. However, the same displacement law in the sigma space, with metric g_{ij}, leads to the relation

$$dl' = +\_d\sqrt{g'_{ij}\xi^i\xi^j} = +\_d\sqrt{fg_{ij}\xi^i\xi^j}$$
$$= l\frac{\partial\sqrt{f}}{\partial x^k}dx^k$$
$$= +_-\frac{1}{2}\frac{\partial lnf}{\partial x^k}dx^k l'.$$

(2.38)

Thus ± (1/2)(∂lnf/∂x^k) plays the role of φ_k in Eqn. (2.36). It follows then that the ordinary connections -($^i{}_{jk}$) constructed from $_{ij}$ are equal to the more general connections $^i{}_{jk}$ constructed according to Eqn. (2.37) from g_{ij} and φ_k = (1/2)(∂lnf/∂x^k): This can also be seen by direct computation from Eqn. (2.36)

$$g'_{ij} = fg_{ij}. \qquad (2.39)$$

and

$$\Gamma'^i{}_{jk} = \Gamma^i{}_{jk}.$$

(2.40)

We may interpret the change of metric from $_{ij}$ to g_{ij} by Eqn. (2.40) as a change of scale for the length at every point of the Riemannian manifold by the variable gauge factor f. This transformation is called a gauge transformation, and φ_k is called a gauge vector field.

The generalized geometry thus separates the problem of measurement of angles from that of measurement of length. For instance, the angle between the two vectors ξ^i and η^i at a given point of the space is measured by the ratio

$$\frac{\xi^i\eta_i}{||\xi||\,||\eta||} = \frac{g_{ij}\xi^i\xi^j}{\sqrt{(g_{ij}\xi^i\xi^j)(g_{ij}\eta^i\eta^j)}}.$$

This ratio does not change under the gauge transformation Eqn. (2.40). The gauge transformation is therefore an angle-preserving, or conformal, change of metric. On the other hand, the length of vectors will change under Eqn. (2.40) according to Eqn. (2.35). Thus the metric tensor $_{ij}$ determines angles, while one needs also the gauge vector φ_k to measure length.

Considering the sigma space, which is characterized by the tensor field $_{ij}$ and gauge vector $_k$. The same argument as before shows that we may, without changing the intrinsic geometric properties of vector fields, replace the geometric quantities by use of a scalar field f as follows:

$$g'_{ij} = fg_{ij}, \quad \phi'_k = \phi_k + \frac{1}{2}\frac{\partial lnf}{\partial x^k}, \quad \Gamma'^i_{jk} = \Gamma^i_{jk}. \tag{2.41}$$

That is, in the new metric, vectors will have the same law of affine transplantation and the angle between different vectors at the same point of the manifold will be preserved, but the local lengths of a vector will be changed according to

$$l'^2 = fl^2.$$

Thus the general Weyl geometry of the sigma space admits also a conformal gauge transformation.

D. Mechanical Systems Near Equilibrium

2.10 Special Relativistic and Classical Mechanics

Classical mechanics describes the motion of a system, which could be a particle, for which the energy of the system is a constant. The equations of motion yield trajectories resulting from the action of forces; they may also be obtained from the Principle of Least Action. When the action integral is treated as a variational problem with variable end points, the method of Lagrangian multipliers yields the same equations as does Hamilton's Principle. However, if the variational problem is transformed to a new space in which the new variational problem has fixed end points, then the metric for this space is displayed, and the equations of motion are geodesics in this space.

In classical mechanics the Principle of Least Action as formulated by Lagrange has the integral form

$$A = \int_{P_1}^{P_2} m\bar{v} \bullet d\bar{s}. \tag{2.42}$$

In curvilinear coordinates the integral assumes the form

$$A = \int_{P_1}^{P_2} mg_{\alpha\beta}\frac{dx^\alpha}{dt}dx^\beta = \int_{t(P_1)}^{t(P_2)} mg_{\alpha\beta}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt}dt,$$

where $\alpha,\beta = 1, 2, 3$.

Defining

$$T = \frac{m}{2}g_{\alpha\beta}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt},$$

the integral becomes

$$A = \int_{t(P_1)}^{t(P_2)} 2T dt.$$

Then the principle of least action may be stated as:

Of all curves C' passing through P_1 and P in the neighborhood of the trajectory C, which are traversed at a rate such that, for each C', for every value of t, T+V=F, that one for which the action integral A is stationary is the trajectory of the particle.

The transformation of variables may be carried out to display the metric

$$(ds)^2 = h_{\alpha\beta}\, dx^\alpha\, dx^\beta \tag{2.43}$$

where $h_{\alpha\beta} = 2m(E_0 - V)g_{\alpha\beta}$. Here different particles in the same field and with different energies E_0 would appear to have different geometries, a situation which has been previously taken to be impossible and therefore precluded the geometrization of dynamics(see page 6 of ref. 46). However, in view of Weyl's generalization of geometry, treating the variational problem in the Principle of Least Action as transformed to a new space in which the variational problem has fixed end points, in effect, is a transformation into a space with Weyl geometry where the gauge function is $2m(E_0-V)$. Thus changing the energies does not change the geometry since it will still be a Weyl space.

Suppose now that the concepts of classical mechanics are compared with the concepts from the point of view of the Dynamic Theory. The energy of the system in classical mechanics is a constant of the motion and therefore the change in kinetic energy is the negative of the change in potential energy, which may be written as

$$dH = dT + dV = 0.$$

However, for classically conservative forces dH is a perfect differential. Therefore for this system with only one work term the force is a function of position only.

This suggests the association of the classical energy of the system, H, with the system energy, U, which is also a perfect differential. Now if the system is isolated, or E-conservative, then

$$0 = \_\overline{E} = d\overline{U} - \overline{F}dq.$$

But if $dU = dH = 0$ then F must be zero. This points out an important difference between classical physics and the Dynamic Theory. A classically conservative system is one for which the system's energy is a constant of the motion. However, the *E*-conservative system, within the Dynamic Theory, is one for which dE = 0. Thus an *E*-conservative system which is also conservative in the classical sense must have no forces *F* which may depend upon velocity as well as position but may have forces which arise from $-(\partial U/\partial q) = F$ and must be functions of positions only.

Suppose we now turn our attention to the mechanics of Special Relativity. In the Special Theory of Relativity Einstein sought to put Newtonian mechanics into a form that would leave the speed of light invariant. The resulting dynamics exhibits the notion of a unique velocity in a similar sense to the previously defined absolute velocity.

Within the Dynamic Theory we may display the appearance of the Special Theory's foundations by using the generalized entropy principle rather than being required to assume the existence of Newton's equations of motion on an a priori basis.

Newtonian mechanics is displayed in its simplest form for particles, so we shall make the restrictive assumption that the mass density, γ_0, such that

$$\int \gamma_0 \, d(vol) = m_0$$

We will also assume that the g_{ij} are constants, thus

$$(d\sigma)^2 = c^2 (dt)^2 - \hat{g}_{\alpha\beta}\, dx^\alpha dx^\beta , \quad \alpha, \beta = 1,2,3,$$

and $g_{\alpha\beta}=\delta_{\alpha\beta}$. Our variational problem depends upon the integral

$$S = \int_0^S dS = \int_0^S \sqrt{(dS)^2} = \int_0^{mq^0} \sqrt{m_0{}^2 (dq^0)^2} = \int_{\sigma_1}^{\sigma_2} \sqrt{m_0^2 f\, \hat{g}_{jk} u^j u^k}\, d\sigma ,$$

where we have used the definition

$$u^j = \frac{dx^j}{d\sigma}.$$

Because we have assumed that the mass, m_0, is a constant we can write our integral as

$$q^0 = \frac{S}{m_0} = \int_{\sigma_1}^{\sigma_2} \sqrt{f\, \hat{g}_{jk} u^j u^k}\, d\sigma .$$

We can make a change of variables by letting

$$ds = \sqrt{\hat{g}_{jk} u^j u^k}\; d\sigma , \tag{2.44}$$

so that the integral becomes

$$q^0 = \int_{S_1}^{S_2} \sqrt{f}\, dS.$$

We can now define a new function

$$T = \frac{f}{2} m_0 c^2,$$

and then consider a further change in variables such that

$$dS = c\sqrt{f}, \quad d\tau = \sqrt{\frac{2T}{m_0}}\, d\tau.$$

If we substitute this new variable into our integral we find

$$m_0 c q^0 = \int_{\tau(P_1)}^{\tau(P_2)} 2T\, d\tau, \tag{2.45}$$

with the auxiliary selection that $2T - m_0c^2f = 0$.

The problem of determining geodesics has now been converted into a statement of the principle of maximum generalized entropy:

> Of all curves C', passing through P_1 and P_2 in the neighborhood of the trajectory C, which are traversed at a rate such that, for each C', and for every value of τ, $2T = m_0c^2f$, that one for which the generalized entropy integral, q^0, is maximum (stationary) is the trajectory of the particle.

When stated in the form of a variational equation, this principle reads

$$\delta \int_{\tau(P_1)}^{\tau(P_2)} 2T\, d\tau = 0,$$

with the auxiliary condition $2T-m_0c^2f=0$, on C'.

The dynamics is unaffected by the addition of a constant to the gauge function; therefore, let

$$V = h - m_0 c^2 \frac{f}{2},$$

where h is a constant. The auxiliary condition now reads

$$T + V - h = 0, \text{ on } C'.$$

We can solve this variational problem by making use of the Lagrangian multiplier method for a problem with constraints. We construct a function G=2T+λφ, where φ=T+V-h=0, and determine the solution of the system of equations

$$\frac{\partial G}{\partial x^j} - \frac{d}{d\tau}\left[\frac{\partial G}{\partial u^j}\right] = 0, \quad j = 0,1,2,\bullet\bullet\bullet,n, \text{ with}$$
$$T + V - h = 0.$$

(2.45)

This system has a solution for which λ=-1[Sokol], and it follows that the trajectory C is determined by the solution of the system

$$\frac{d}{dt}\left[\frac{\partial T}{\partial u^j}\right] - \frac{\partial T}{\partial x^j} = -\frac{\partial V}{\partial x^j}, \quad j = 0,1,2,\bullet\bullet\bullet,n.$$

(2.46)

We assumed that the g_{jk} were constants; therefore, if we make the definition

$$F_j = -\frac{\partial V}{\partial x^j},$$

then the equations in Eqn. (2.46) become

$$\frac{d}{d\sigma}\left[m_0 c^2 \hat{g}_{jk} u^k\right] = F_j,$$

because dσ=cdτ and $\tau = m_0 g_{jk} u^j u^k$. If we multiply these equations by g^{li} and sum them, we obtain

$$\frac{d}{d\sigma}\left[m_0 u^l\right] = \hat{g}^{lj} F_j = F^l, \quad l = 0,1,2,\bullet\bullet\bullet,n.$$

(2.47)

From the metric with constant coefficients we get

$$\frac{d\sigma}{dt} = \sqrt{c^2 - \hat{g}_{\alpha\beta}\dot{x}^\alpha \dot{x}^\beta}, \quad \alpha,\beta = 1,2,3,$$

or

$$\frac{d\sigma}{dt} = \sqrt{c^2 - v^2}.$$

(2.48)

Substituting Eqn. 2.48 into Eqn. 2.47 we find that

$$F^l = \frac{d}{d\sigma}\left[m_0 c^2 u^l\right] = \frac{d}{dt}\left[m_0 c^2 \frac{dx^l}{d\sigma}\right]\frac{dt}{d\sigma}$$

$$= \frac{1}{\sqrt{c^2 - v^2}}\frac{d}{dt}\left[m_0 c^2 \frac{dx^l}{dt}\left(\frac{dt}{d\sigma}\right)\right],$$

$$= \frac{1}{\sqrt{c^2 - v^2}}\frac{d}{dt}\left[\frac{m_0 c^2}{\sqrt{c^2 - v^2}}\right]v^l$$

where $v^l = dx^l/dt$. Thus we have

$$F^l = \frac{1}{\sqrt{1-\beta^2}}\frac{d}{dt}\left[\frac{m_0 v^l}{\sqrt{1-\beta^2}}\right], \tag{2.49}$$

where $\beta = v/c$. Now Eqn. (2.49) can be rewritten as

$$\sqrt{1-\beta^2 F^l} = \frac{d}{dt}\left[\frac{m_0 v^l}{\sqrt{1-\beta^2}}\right]. \tag{2.50}$$

Because $\beta=0$ in a local coordinate system x,

$$F^l = m_0 \frac{d^2 x^l}{dt^2} = m_0 a^l, \tag{2.51}$$

where, in a local reference frame x, $a^l = (1/c^2)(d^2x^l/dt^2)$. In Eqn. (2.51) we have the form of Newton's second law in classical mechanics.

We may rewrite Eqn. (2.50) in the form

$$\sqrt{1-\beta^2}\, F^l \equiv f^l = \frac{d}{dt}\left[\frac{m_0 v^l}{\sqrt{1-\beta^2}}\right]. \tag{2.52}$$

These equations are the equations of motion of the Special Theory of Relativity and come from the geodesic equations of the variational problem, in Eqn. (2.42), based upon the generalized entropy principle with the restrictive assumptions that the mass density, γ, be a constant and that the metric coefficients, g_{jk}, are independent of the mass distribution. Thus, we have shown that the special relativistic equations of motion and the Newtonian equations of motion are required by the generalized entropy principle, but that they represent a limited subset of the entropy principle.

2.11 Energy Concepts

Newtonian and relativistic mechanics talks of potential and kinetic energy while classical thermodynamics, which forms the basis of the Dynamic Theory, contains concepts with units of energy such as entropy, enthalpy, and free energy. We may use these common fundamental principles within the Dynamic Theory to explore how the mechanical energy concepts fit among the general thermodynamic energy functions. It seems that this will be of more than a little benefit when trying to keep all of the energy-based concepts in proper perspective.

First, let us recall the First Law, with n=1,

$$\_E = dU - Fdx, \tag{2.53}$$

whereas the differential change in the generalized entropy is

$$dS = \frac{dU}{\phi} - \frac{Fdx}{\phi}, \tag{2.54}$$

where dS is a total differential. If we suppose that the system energy, U, may be a function of position, x, and the velocity, v=dx/dt, then we may write

$$dS = \frac{1}{\phi}\frac{\partial U}{\partial v}dv + \frac{1}{\phi}\frac{\partial U}{\partial x}dx - \frac{F}{\phi}dx.$$

Because dS is a total differential, then

$$\frac{\partial}{\partial v}\left\{\frac{1}{\phi}\left[\frac{\partial U}{\partial x} - F\right]\right\} = \frac{\partial}{\partial x}\left\{\frac{1}{\phi}\frac{\partial U}{\partial v}\right\}.$$

This requires that

$$\left(\frac{1}{\phi}\right)\frac{\partial^2 U}{\partial x \partial v} -$$

(2.55)

because φ=φ(v) from the second law. Further, dU is a total differential so Eqn. (2.55) becomes

$$\frac{\frac{\partial F}{\partial v}}{\left[\frac{\partial U}{\partial x} - F\right]} = -\frac{\frac{\partial \phi}{\partial v}}{\phi} = \textit{function of velocity only.}$$

Now consider the functional form of the force from the equations of motion in Eqn. (2.52),

$$F = \sqrt{1-\beta^2}\,\hat{F}(x) \equiv \Phi\hat{F}(x),$$

(2.56)

where F(x) is strictly a function of x because it came from the gauge function. Then

$$\frac{\partial F}{\partial v} = \frac{d\Phi}{dv}\hat{F}(x).$$

Thus, Eqn. (2.55) may be written as

$$\frac{\hat{F}(x)\frac{d\Phi}{dv}}{\left[\frac{\partial U}{\partial x} - \Phi\hat{F}(x)\right]} = -\frac{\frac{d\phi}{dv}}{\phi}.$$

(2.57)

In order to satisfy Eqn. (2.57) we find Φ=φ and U=U(v).

By substituting these results into the differential expression for the entropy, Eqn. (2.54) we find

$$dS = \frac{\left(\frac{dU}{dv}\right)dv}{\sqrt{1-\beta^2}} - \hat{F}(x)\,dx,$$

(2.58)

which is a perfect differential whereby we have found that U is strictly a function of velocity.

Now consider the First Law for an isolated system, or

$$\_E = 0 = dU - Fdx,$$

but, using Eqn. (2.56) this may be written as

$$dU = \sqrt{1-\beta^2}\, F(x)\, dx.$$

Then by integrating we find that

$$U - U_0 = \int_{P_0}^{P} F\, dx.$$

This is Einstein's energy integral which, because of the equations of motion, becomes

$$U = \frac{m_0 c^2}{\sqrt{1-\beta^2}} + constant. \tag{2.59}$$

In his Special Theory of Relativity Einstein interpreted the right-hand side of Eqn. (2.59) to be the kinetic energy; therefore, he chose the integration constant to be $-m_0c^2$ in order that T=0 when v=0. Here, Eqn. (2.59) is the energy of the system and, therefore, will not be zero when v=0. Thus, the constant of integration should be taken as zero, giving the energy by

$$U = \frac{m_0 c^2}{\sqrt{1-\beta^2}}. \tag{2.60}$$

If we differentiate Eqn. (2.60) with respect to the velocity, we find

$$\frac{\partial U}{\partial v} = \frac{\frac{1}{2} m_\eta c^\theta \frac{2v}{c^\theta}}{(1-\beta^\theta)^{\frac{I}{\theta}}} = \frac{m_\eta v}{(1-B^\theta)^{\frac{I}{\theta}}}.$$

Substituting this result into Eqn. (2.58), the change in entropy becomes

$$dS = \frac{m_0 v dv}{(1-\beta^2)^{2}} - \hat{F}(x)dx.$$

This expression may now be integrated because

$$S - S_\eta = \int_\eta \frac{m_\eta v}{(1-\beta^\theta)^\theta} dv - \int_1^2 \hat{F}(x)dx$$

$$= \frac{1}{2} m_\eta c^\theta \int_\eta \frac{d(\beta^\theta)}{(1-\beta^\theta)^\theta} + V(x),$$

and

$$S = \frac{\frac{1}{2} m_0 c^2}{(1-\beta^2)} + V(x) + constant.$$

By setting the constant of integration at 1/2 m_0c^2, we get

$$S = \frac{\frac{1}{2} m_0 c^2 \beta^2}{(1-\beta^2)} + V(x) \quad or \quad S = \frac{\frac{1}{2} m_0 v^2}{(1-\beta^2)} + V(x). \tag{2.61}$$

Thus, the generalized entropy for a purely mechanical system has two parts. One, depending entirely upon the velocity, and which we may call kinetic entropy, is given by

$$T = \frac{\frac{1}{2} m_0 v^2}{(1-\beta^2)}. \tag{2.62}$$

The second term in the mechanical entropy is a function of position only and may be called the entropy potential, V(x).

We may look at the kinetic entropy differently if we go back to the variable changes during the presentation of the maximum entropy principle, because there we had

$$ds = \sqrt{\hat{g}_{jk} u^j u^k}\, d\sigma = \sqrt{\frac{2t}{m_0}}\, d\tau,$$

but $d\sigma = cd\tau \sqrt{1 - v^2/c^2}$ 199 therefore,

$$T = \frac{1}{2} m_0 c^2 \hat{g}_{jk} u^j u^k = \frac{1}{2} m_0 c^2 \hat{g}_{jk} \frac{du^j}{d\sigma} \frac{du^k}{d\sigma}$$

$$= \frac{1}{2} m_0 c^2 \hat{g}_{jk} \dot{x}^j \dot{x}^k \left(\frac{dt}{d\sigma} \right)^2.$$

But, by Eqn. (2.48) this becomes

$$T = \frac{\frac{1}{2} m_0 v^2}{(1 - \beta^2)},$$

which is the kinetic entropy of Eqn. (2.62). Thus, we find that it is the mechanical entropy, S, that must have a constant value along any trajectory for an isolated system, because

$$T + V - h = 0$$

for the trajectory, and therefore, S=h=constant.

Thus we have established the following for the trajectories of an isolated system:

Mechanical Entropy: $S = h = constant$

Kinetic Entropy : $T = \frac{m_0 c^2}{(1 - \beta^2)}$

Potential Entropy : $V(x) = -\int \hat{F}(x)$

Energy of the system : $U = \frac{m_0 c^2}{\sqrt{1 - \beta^2}}$

Work done by the system : $W = \int_{x_1}^{x_2} F_j dx^j$

Kinetic energy of the system : $T = m_0 c^2 \left[\frac{1}{\sqrt{1 - \beta^2}} - 1 \right]$

Force : $F_j = \sqrt{1 - \beta^2} \hat{F}_j(x)$

Rest energy of the system : $U(v = 0) = m_0 c^2$

First Law : $\bar{d}E = 0 = dU - F_j dx^j$,

or $0 = \frac{m_0 v dv}{(1 - \beta^2)^{\frac{3}{2}}} - \sqrt{1 - \beta^2} \hat{F}_j dx^j$

2.12 Non-Isolated System

Thus far we have consistently required the system to be isolated. Obviously there are a large number of physical phenomena for which this restriction may not be used, even as an approximation. Therefore, relaxation of this restriction should provide description of a large and important class of systems. The remainder of this book involves the investigation of the predictions of the Dynamic Theory assuming the system is isolated. This may give the implication the non-isolated system is less important or less interesting. This is not the intention of the presentation. Rather, the presentation is aimed at displaying the fact that existing theories are subsets of the Dynamic Theory. In order to do this we must stay with the assumption of the isolated system.

One of the benefits of the Dynamic Theory is the capability of using procedures currently used in one branch of physics in another where prior to the unification displayed here would have been thought impossible. A system in which this procedure should produce significant results is a nonequilibrium thermodynamic system. Thermodynamics tells us that we must minimize the free energy, but the ability to use this as a variational principle to obtain equations of motion is a procedure that the Dynamic Theory now makes possible for this thermodynamic system.

E. Quantum Mechanics

2.13 Quantum Mechanics Derived

In 1927 F. London derived quantum principles from Weyl's geometry.(47) However, the results of his work made it difficult to define length as a real number and because of this Weyl later interpreted the mathematical formalism of his unified theory as connected with transplanting a state vector of a quantum theoretical system.

Suppose that we consider an isolated, or E-conservative, system so that dE = 0. Then, because of the Second Law $dq^0 \geq 0$ which is the principle of increasing mechanical entropy. Then certainly $(dq^0)^2 \geq 0$ and also, since $(dq^0)^2 = f(d\sigma)^2$, then $f(d\sigma)^2 \geq 0$. However, if $f<0$, then $(d\sigma)^2<0$ since it is the product that must remain greater than, or equal to, zero. In this case

$$dq^0 = \sqrt{-f}\sqrt{-(d\sigma)^2}.$$

But

$$d(d\sigma) = \phi_k dx^k (d\sigma)$$

and

$$\int \frac{d(d\sigma)}{(d\sigma)} = \int \phi_k dx^k,$$

which implies that the element of arc $(d\sigma)$ is given by

$$(d\sigma) = (d\sigma)_0 e^{\int \phi_k dx^k},$$

where $(d\sigma)_0$ is some initial value of the element of arc.

Now suppose an equilibrium, or reversible, state is desired so that $dq^0=0$. Thus, the desired condition is a null trajectory of the $(dq^0)^2$

manifold. Then, if $f \neq 0$, the desired condition is also a null trajectory of the $(d\sigma)^2$ manifold. This implies that $d(d\sigma)=0$ or $(d\sigma)=(d\sigma)_0$, so that

$$e^{\int \phi_k dx^k} = 1,$$

which is satisfied only if

$$\int \phi_k dx^k = 2\pi i N, \tag{2.63}$$

where N is an integer. This is the quantum condition London introduced.

To illustrate how this condition arises from the Dynamic Theory's approach, suppose a description of a hydrogen atom is desired. A hydrogen atom is in a stable condition and, if isolated, satisfies the conditions dE = 0 and $dq^0=0$. These conditions along with $f \neq 0$ establish the quantization of the integral in Eqn. (2.63). To show how the Dynamic Theory removes from London`s work the difficulty of defining length as a real number, consider an elementary presentation of London's. Suppose the field of a proton to be given by

$$\phi_0 = \frac{\alpha^1}{r}; \;\; \phi_i \equiv 0; \;\; i \neq 0.$$

Equality of forces for the simple case of circular motion requires that

$$\frac{mv^2}{r} = \frac{e^2}{r^2}.$$

Thus the period is given by $T=2\pi r/v$ and the velocity by

$$v = \frac{e}{\sqrt{mr}}.$$

Now

$$\int \phi_k dx^k = \int \phi_0 cT = 2\pi i N,$$

so that

$$\frac{\alpha^1 cT}{r} = \alpha^1 c 2\pi \sqrt{\frac{mr}{e}} = 2\pi i N.$$

Solving for the radius shows that the allowed radii are

$$r = \frac{-N^2 e^2}{(\alpha^1)^2 m c^2}.$$

By choosing

$$\alpha^{1} = \frac{2\pi i e^{2}}{hc} \approx \frac{i}{137} \equiv i\alpha,$$

where h is Planck's constant, then the possible radii become

$$r = \frac{N^{2} h^{2}}{4\pi^{2} e^{2} m},$$

which are the Bohr radii.

The imaginary α^1 presented the difficulty, in London's work, of defining length as a real number. In the Dynamic Theory real distance, or length, may be defined, and properly should be, in the (dq^0) manifold. Recalling that the definition of the potentials is

$$\phi_k = \pm \frac{\partial \ln f^{\frac{1}{2}}}{\partial x^k},$$

it may easily be seen that if f<0, then φ_k becomes imaginary as does the length of arc in the $(d\sigma)^2$ manifold since the length of arc is given by

$$\sigma = \int \sqrt{(d\sigma)^{2}}.$$

However, the arc length in the $(dq^0)^2$ manifold is real since $dq^0 \geq 0$ by the Second Law.

It should be noted that the conditions for quantization are not restricted to dE=0, $dq^0=0$, and f<0 as used here. Any set of conditions which results in the final element of arc $(d\sigma)$ being equal to the initial element of arc $(d\sigma)_0$ results in quantum conditions. It is particularly significant to note that the quantization involves only forces, which may be described in terms of the "distance curvature" and does not involve forces describable by a vector curvature. Thus interpreting the gauge potentials φ_k to be electromagnetic potentials provides quantum effects for electromagnetic forces.

Here, again, is a distinction between curved and Euclidean manifolds, though here it appears slightly different. The Dynamic Theory requires a quantization. However, this quantization depends upon the existence of a gauge function and appropriate restrictive conditions. Thus a curved space may exhibit quantum effects but only if the curvature is accompanied by a gauge function or a distance curvature.

Thus the Dynamic Theory, through London's quantization, not only supports the contention that "God does not play with dice all the time" but, further, may supply the answer to the question, "What is waving in the wave function?" London showed that the wave function is directly related to the element of the arc length in the sigma manifold. Therefore the "waving" is the tendency of this element of arc length to increase and decrease around a closed path. Using the calculus of complex variables, the quantum number becomes the order, or multiplicity of the zero of (dσ). This may also be stated in terms of null trajectories. Einstein's null trajectory was the path light travels and remains so here. However, this is the zeroth order null trajectory. The remaining null trajectories for the complex arc length are given, as London showed, by the equations of Quantum Mechanics.

2.14 On The Derivation of Thermodynamics from Statistical Mechanics

It is commonly believed that one can "derive" thermodynamics from a variety of force laws using the techniques of statistical mechanics. This belief is not supported when one considers the development of statistical thermodynamics. For instance, in order to talk of a statistical temperature, τ, one must start by assuming Newtonian physics (this constitutes three fundamental assumptions). Given Newtonian physics one can talk of an energy distribution, canonical ensembles and statistical temperature; however, one must make an additional fundamental assumption (the Equipartition Law) before the statistical heat capacities may be obtained.

In order to obtain thermodynamics we need two more assumptions. To display the assumption necessary for the first law of thermodynamics let me quote from page 85 of "Basic Theories of Physics: Heat and Quanta" by Peter G. Bergmann. "The difference between the heat transferred to the system and the work performed by it,

$$\_Q - \_W = C_v \, d\theta + \sum_{i=1}^{n} (J_i - Y_i) d R_i , \tag{2.64}$$

is, according to our previous discussions, the increase in u. But in a systematically thermodynamic approach (that is, using only macroscopic observations and concepts), we get the differential expression, Eqn. (2.64) without reference to u. From that point of view, to claim that this expression is an exact differential is a logically new assertion; and this assertion constitutes the First Law of Thermodynamics."

The assumptions of statistical thermodynamics allow us to derive the differential of the heat exchanged in the form

$$\_Q = \tau d\sigma , \tag{2.65}$$

where τ is the statistical temperature and dσ is the statistical entropy. Further, it may be shown that

$$d\sigma \geq \frac{\_Q}{\tau}. \tag{2.66}$$

By comparing Eqn. (2.66) with the classical thermodynamic statement

$$\_Q = TdS, \tag{2.67}$$

and

$$dS \geq \frac{\_Q}{T}, \tag{2.68}$$

we find that the statistical expression, Eqn. (2.66), is analogous to the classical expression, Eqn. (2.68), for the second law of thermodynamics. Also we may equate Eqns. (2.65) and (2.67) to obtain

$$\tau d\sigma = TdS, \tag{2.69}$$

and Eqns. (2.66) and (2.68) are simultaneously satisfied provided that $\tau/T>0$. In statistical thermodynamics it is asserted that $\tau/T=k_B$, where k_B is Boltzman's constant. Once this assertion is made then we have $k_B d\sigma = dS$, hence $S=k_B\sigma$. However, there is no logical necessity that the ratio τ/T be a constant from the statistical approach, and only if it is a constant can we have a one-to-one correspondence between the statistical entropy and the classical thermodynamic entropy.

The misconception that classical thermodynamics may be derived from Newtonian mechanics without the necessity of making additional assumptions is further entrenched by authors, such as Kittel, who in his text Thermal Physics says the following on p. 49, "We show in Chapter 8 that τ is proportional to the conventional absolute temperature which is measured in degrees Kelvin"; (This implies a logical necessity) On p. 427 the author states, "By analogy with the relation $dQ = \tau d\sigma$ we `assume' that the Kelvin temperature T has the property $dQ = k_B T d\sigma$ for a reversible process; here k_B is a constant to be determined and σ is the entropy." (The implied logical necessity is reduced to an assumption.)

We are so familiar with Newtonian mechanics and its basic validity that it is difficult for us to consider that it might be derivable from some other physical concept and its associated fundamental assumptions. Further, classical thermodynamics, even before statistical mechanics gave rise to the distinction between microscopic and macroscopic views, never talked of anything resembling equations of motion. Added to these factors is the somewhat long logical progression from the adopted laws of the Dynamic Theory to Newtonian mechanics.

On the one hand the First and Second laws adopted by the Dynamic Theory give rise to a generalized entropy principle that requires that any dynamics for an isolated system must occur so as to seek a maximum of the generalized entropy. Thus we have a variational principle based upon maximizing the entropy. On the other hand the laws produce, through the stability conditions, a metric to be used in this variational principle in which the type of geometry is specified and need not be assumed as in Newtonian and relativistic mechanics. The Euler equations resulting from this variational principle taken in the non-relativistic, Euclidean, three-dimensional limit, for particles become

$$m\frac{d^2x^i}{dt^2} = F_i,$$

or inertial mass, m, times the acceleration, d^2x^i/dt^2, must equal the force, F_i. Thus the adopted laws, through restrictive assumptions, do lead to Newton's Second Law. Newton's First Law comes from considering the motion in the absence of any force. To arrive at Newton's Third Law one must show that all of the forces, allowed by the adopted laws of the Dynamic Theory, must be symmetrical. A violation of this symmetry of forces that has recently been found will be shown in Section 4.11 for forces within the nucleus; and, therefore, the Dynamic Theory does not require Newton's Third Law to hold within the nucleus, but does for atomic forces and macroscopic matter.

F. Summary

2.15 Summary of new theoretical fundamentals

When this investigation was initiated, it was concluded that Einstein's postulate of the constancy of the velocity of light could not be adopted since it was felt that experimental evidence in electromagnetism alone did not justify applying it as a limiting velocity to all types of forces. However, we find that Einstein's postulate is required by the Dynamic Theory which approaches physical phenomena from a different way. The new viewpoint indeed supports Einstein's every contention with regards to Special Relativity and his uneasiness concerning quantization. Further, the Dynamic Theory supports Einstein in such a way that it seems only the early successes of his theories kept Einstein himself from coming to the same realization.

This is, of course, speculation, but it was Einstein who returned to very fundamental concepts in order to establish a basis for his relativity theory. He was also known to be aware of the tremendous strength of classical thermodynamics since he wrote, "A theory is the more impressive the greater the simplicity of its premises are, the more different kinds of things it relates, and the more extended is its area of applicability. Therefore the deep impression that classical thermodynamics made upon me. It is the only physical theory of universal content concerning which I am convinced that, within the framework of applicability of its basic concepts, it will never be overthrown." Thus it seems only the fact that Caratheodory's statement of the second law, which is the key to the development of the Dynamic Theory, did not make its appearance before the relativistic theory had achieved such stupendous successes kept Einstein from eventually investigating its possible extended application.

The key points in the development of the Dynamic Theory seem to be the recognition of the generality of the thermodynamic laws and their independence upon the number or type of variables considered and the recognition that the quadratic forms associated with the stability conditions from natural metrics leading to a geometrical description of the dynamics of the system independent of the variables used in the description.

There are numerous conclusions and implications that could be reiterated here; however, only a few of the seemingly more significant ones will be discussed. The first one is the existence of an integrating factor for any system describable by the First Law, particularly an integrating factor independent of the type of force considered. It is this fact that ultimately leads to a unique limiting velocity for all forces. However, in speaking of the absolute velocity for mechanical systems, care must be taken to point out that, as far as the three laws are concerned, it does not represent an absolute barrier. Rather the laws only state that, for a mechanical system with only three work terms representing the work done by three spatial forces the absolute velocity represents an upper and lower limit. Thus solutions with velocities greater than the speed of light are also allowed. However, so long as the system is subjected to only these three forces, then its velocity may never cross this barrier. This absolute barrier effect may be expected to change if another force term representing an additional dimension is found necessary.

The reduction in the number of fundamental laws or postulates is significant. This together with the unifying effect of the three laws promises to simplify the study of physical phenomena by founding the entire realm of physics upon a common set of conceptualizations.

In this chapter it was shown that the Special Theory of Relativity was a special case obtained from the fundamental laws adopted by the Dynamic Theory. Einstein's postulate concerning the speed of light was an immediate result of the Second Law. Further, it was shown that Quantum Mechanics resulted when stable isentropic states were required. This displays a different relationship between these theories than has previously been envisioned. Here there is no question concerning one being more fundamental than the other. The determining factor is not which is more fundamental, but what restrictions are placed upon the system. For example, if the system is in an "isentropic state" that state is to be determined by the equations of Quantum Mechanics. If the system is not required to be isentropic or otherwise restricted so that the entropy must return to its original value after completing a loop the quantum Mechanics does not describe the system. Rather, in this case one must turn to the equations of Einstein's Special Theory of Relativity.

A further note should be considered here. The equations of motion that have been derived here and the Quantum Mechanic equations of motion which London derived from the isentropic condition describe the system as if "tends" toward an equilibrium. This is the origin of the motion. That is, the tendency to seek a maximum of entropy for the isolated system.

In this chapter there is no clear way to improve our understanding of nuclear phenomena, nor is there any clarification of gravitational effects. Further, ramifications of the theory will be pursued in the next three chapters, which will bear on these points.