CHAPTER 5 - GRAVITATION

5.1 Charge-to-Mass Ratio and Magnetic Moments

In Chapter 1 a brief overview of the Dynamic Theory was presented. The fundamental principles of the Dynamic Theory were presented in Chapter 2. From these fundamental laws the constancy of the speed of light was derived, the required geometry was obtained, classical and special relativistic equations of motion were derived, and the conditions requiring quantum mechanics were displayed. The requirements of the fundamental laws were carried further in Chapter 3 by looking at the gauge fields of the resulting five-dimensional geometry when mass is considered as an independent variable. When quantization conditions are considered in five-dimensions we found experimental features of particle physics required by these new laws. For example, we saw that octets are required fundamental states reminiscent of Gell-Man's eight-fold way; the allowed fields for fundamental particles were shown to be quantitized in electric charge; and the radial field dependence to display a short-range non-singular behavior which allowed it to predict nuclear masses from it's deviation from the Coulombic radial dependence and nuclear decay (beta decay) from the asymmetry of while particle forces.

Thus, in chapters 2, 3, and 4 we have shown how the Dynamic Theory reproduces, by using the appropriate restrictive assumptions, the fundamentals of all the current branches of physics except gravitation. In Chapter 4 the radial field dependence was derived and the long-range dependence required that the new field components be interpreted as the gravitational field and the gravitational potential. In this chapter we will explore a few aspects of this interpretation. In particular, we will look at some of the predictions of the Dynamic Theory in comparison with Einstein's General Theory of Relativity.

Before we plunge into the derivation of a prediction to compare with the General Theory of Relativity let us first consider a question which arises from the necessity of keeping the units straight among the field quantities E, B, V, and V_4 when they are all to be considered as components of the five-dimensional gauge field. By considering the units of these field components it is soon found that a charge-to-mass ratio is needed in order that the units of the gravitational field components may be compared, or put in the same equation, with the electric and magnetic components. Let us first see if we can determine this ratio.

In the Chapter 4 the derivation of the fields allowed for fundamental particles was presented. These field expressions give rise to the specification of a charge-to-mass ratio which allows conversion of classical gravitation field units to electromagnetic field units.

The gravitational field component in the system of field equations with the electric and magnetic components brings up the requirement for a gravitation-to-electromagnetic unit conversion. This need may be seen by looking at the different field quantities. First consider the electric case. The field units are given by [E] = volt/meter, while the expression for the electric force density is $F_e = \rho E$ with units of newton/meter^3.

For the gravitational field, in the Dynamic Theory, the units are [V] = webers/meter squared, while the gravitational "current" density has units given by $[J_4]$ = ampere/meter^2. Thus, the gravitational force density is given by $F_a = (J_4/c)V$ where again the units are newton/meter^3.

In order to compare this system to the classical gravitational system we need to be able to go from a gravitational field with units of newton/kilogram to units of volt/meter. Now

$$\frac{nt}{kg} = \frac{volt\text{ - }coul}{m\text{ - }kg} = \left(\frac{volt}{m}\right)\left(\frac{coul}{kg}\right)$$

Thus, if ß is a quantity with units of coul/kg, then (1/ß) is the conversion factor we seek. Similarly, we need to convert the gravitational mass density, (J_4/c), with units of coul/m^3 to units of kg/m^3. Obviously ß will also be the conversion factor for this also. The question is; How do we determine this charge-to-mass ratio and is it unique?

If we consider the fields the Dynamic Theory gives for fundamental particles we may determine ß. Thus, let us look at the solution of the gauge function for fundamental particles given previously, that is

$$ln\, F^{\frac{1}{2}} = f_r f_\theta f_\phi f_\gamma f_t \quad .$$

We showed that this became

$$ln\, f^{\frac{1}{2}} = f_\gamma f_t \left[\frac{e^{\left(-\frac{\lambda}{r}\right)}}{r}\right] \tag{5.1}$$

for fundamental particles. The functional dependence of the gauge function upon time or mass density was not determined then.

Recalling from past reading that measurements of a time dependence of the earth's gravitational field have been reported(). We may proceed to make the simplest possible assumption about the functional form for f_γ and f_t, namely linear dependence. If the functions have only weak dependence upon time or mass density then this dependence could easily be masked in experimentation not specifically designed to look for it. Thus, lets' consider the form

$$ln\, f^{\frac{1}{2}} = (a + bt)(s + w\gamma)\left[\frac{e^{\left(-\frac{\lambda}{r}\right)}}{r}\right] \tag{5.2}$$

where a, b, s, and w are constants to be evaluated using known information about the proton and time dependence measurements of the gravitational field.

The gauge potentials are then

$$\phi_0 = \frac{\partial(\ln f^{\frac{1}{2}})}{\partial(ict)} = \left(\frac{N_0}{ic}\right) b\,(s + w\gamma)\left(\frac{e^{-\frac{\lambda}{r}}}{r}\right)$$

$$\phi_1 = \frac{\partial(\ln f^{\frac{1}{2}})}{dr} = -\,N_1(a+bt)\,(s+w\gamma)\left(1-\frac{\lambda}{r}\right)\left(\frac{e^{-\frac{\lambda}{r}}}{r^2}\right) \tag{5.3}$$

$$\phi_2 = \phi_3 = 0$$

and

$$\phi_4 = a_0 \frac{\partial(\ln f^{\frac{1}{2}})}{\partial\gamma} = a_0\, w(a+bt)\left(\frac{e^{-\frac{\lambda}{r}}}{r}\right) \ .$$

Using these potentials the field quantities become

$$E_r = \frac{(N_1 - N_0)}{c}\, b(s + w\gamma)\left(1-\frac{\lambda}{r}\right)\left(\frac{e^{-\frac{\lambda}{r}}}{r^2}\right) \ ,$$

$$E_\theta = E_\phi = 0 \ ,$$

$$B_r = B_\theta = B_\phi = 0 \ , \tag{4.4}$$

$$V_r = (N_1 - N_4)\, a_0\,(a+bt)\, w\left(1-\frac{\lambda}{r}\right)\left(\frac{e^{-\frac{\lambda}{r}}}{r^2}\right) \ ,$$

$$V_\theta = V_\phi = 0 \ ,$$

and

$$V_4 = \frac{a_0(N_4 - N_0)}{c}(bw)\left(\frac{e^{-\frac{\lambda}{r}}}{r}\right) \ .$$

In Eqns. (5.4) we may see the effect of the time and mass density dependence of the fields. First, notice that the electric field, E_r, vanishes as the quantity b vanishes. From the expression for the radial component of the gravitational field, V_r. we see that b is the time dependence of the gravitational field. Thus, in order for an electric field to exist there must be a time dependence of the gravitational field. Similarly, one may see that the electric field must depend upon the mass density in order for there to be a gravitational field. For a gravitational potential, V_4, to exist not only must the gravitational field depend upon time but also the electric field must depend upon the mass density. This is a rather extraordinary revelation!

Now we must check these field quantities in the eight field equations. Because B = 0, then

$$\bar{\Delta} \bullet \bar{B} = 0$$

is satisfied. Next

$$\frac{1}{c}\frac{\partial \bar{B}}{\partial t} + \bar{\Delta}\, x\, \bar{E} = \bar{0}$$

implies

$$\bar{\Delta}\, x\, \bar{E} = \bar{0}$$

which is satisfied by the spherical symmetry of the E field. Then

$$\bar{\Delta}\, x\, \bar{B} - \left(\frac{\mu\varepsilon}{c}\right)\frac{\partial \bar{E}}{\partial t} = \left(\frac{\mu 4\pi}{c}\right)\bar{J} - a_0 \frac{\partial \bar{V}}{\partial \gamma}$$

is satisfied if the current density vector, J, vanishes as it should for particles.

Looking at

$$\bar{\Delta} \bullet \bar{E} = \frac{4\pi\rho}{\varepsilon} - a_0 \frac{\partial V_4}{\partial \gamma}$$

we find that this is satisfied by a spatial charge distribution of

$$\rho = \left(\frac{\varepsilon}{4\pi}\right)\frac{(N_1 - N_0)b(s + w\gamma)\lambda\left(2 - \frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}}}{cr^4} \qquad (5.5)$$

The continuity equation

$$0 = \frac{\partial \rho}{\partial t} + \bar{\Delta} \bullet \bar{J} + a_0 \frac{\partial J}{\partial \gamma}$$

requires

$$\frac{\partial J_4}{\partial \gamma} = 0 \quad . \qquad (5.6)$$

Then

$$\bar{\Delta}\, x\, \bar{V} + a_0 \frac{\partial \bar{B}}{\partial \gamma} = 0$$

implies

$$\overline{\Delta}\, x\, \overline{V} = 0$$

which is also satisfied by the spherical symmetry of V.

When we consider the radial component of

$$\frac{1}{c}\frac{\partial \overline{V}}{\partial t} + \overline{\Delta} V_4 = a_0 \frac{\partial \overline{E}}{\partial \gamma}$$

we find that it is satisfied if

$$N_4 = N_0 \quad , \tag{5.7}$$

while the components in the and directions are satisfied identically.

The last equation is

$$\overline{\Delta} \bullet \overline{V} + \left(\frac{\mu\varepsilon}{c}\right)\frac{\partial V_4}{\partial t} = -\left(\frac{4_\pi\, \mu}{c}\right) J_4 \quad .$$

This equation requires that

$$J_4 = -\left(\frac{a_0 c w}{4\pi\mu\, r^4}\right)(N_1 - N_0)(a + bt)\lambda\left(2 - \frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}} \quad . \tag{5.8}$$

Note that the expression for J_4 satisfies Equation (5.6). We also find that the radial dependence of J_4 is identical to the radial dependence of ρ. Thus, we may rewrite Eqn (5.8) as

$$J_4 = \left[\frac{-a_0 c^2 (a + bt) w}{\mu\varepsilon b(s + w\gamma\ )}\right]\rho.$$

T.C. Van Flandern, of the Naval Observatory, has reported a measured very small time rate of decrease in the gravitational field given by him to be approximately 6-parts in 10^{11} per year. If we designate this rate of decrease by dG/dt, then

$$\frac{b}{a} = \dot{G} \quad . \tag{5.9}$$

From the experimental measurement

$$\dot{G} \cong .6\ x\ 10^{-11}/yr \cong -1.9\ x\ 10^{-18}\ \sec^{-1} \quad ,$$

then

$$a + bt \cong a \quad .$$

Thus, the non-zero field quantities are

$$E_r = a\dot{G}z(s + w\gamma)\left[\frac{\left(1 - \frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}}}{r^2}\right] ,$$

$$V_r = Z\, a_0\, w(a + bt)\left[\frac{\left(1 - \frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}}}{r^2}\right]$$

$$\cong Z\, a_0\, wa\left[\frac{\left(1 - \frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}}}{r^2}\right] , \qquad (5.10)$$

$$\rho = \frac{-Z\varepsilon\dot{G}a(s + w\gamma)\lambda\left(2 - \frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}}}{4\pi\, cr^4}$$

$$J_4 = \left[\frac{-a_0 c^2 aw}{\mu\varepsilon b(s + w\gamma)}\right]\rho$$

where $Z = N_1 - N_0$.

We should note in Eqns. (5.10) that the gravitational field depends upon the universal constant, a_o, being non-zero. This implies a linkage between the maximum mass conversion rate and the gravitational field in somewhat a similar way as the electric field depends upon the speed of light. Further, it should be noted that if the electric field does not depend upon the mass then there could be no gravitational field.

Now the total charge is given by

$$q = \int_{vol} \rho ds \quad .$$

Therefore, using Eqn. (5.5) and the spherical element of volume $dv = r^2\sin\theta\, dr d\theta d\varphi$, we have

$$q = \int_{vol}\left(\frac{-Z\varepsilon a\dot{G}}{4\pi c}\right)\lambda\,(s + \omega\gamma)\left(2 - \frac{\lambda}{r}\right)\frac{e^{-\frac{\lambda}{r}}}{r^4}(r^2\sin\theta dr d\theta d\phi) \quad .$$

$$= \left(\frac{-Z\varepsilon a\dot{G}\lambda}{4\pi c}\right)\left\{\int_{r=0}^{r=R}(s + w\gamma)\left(2 - \frac{\lambda}{r}\right)\left[\frac{e^{-\frac{\lambda}{r}}}{r^2}\right]\sin\theta dr d\theta d\phi\right\}$$

where R is the radius of physical extent of the particle. If the mass density in the particle is a constant, γ_0, then the charge is given by

$$
\begin{aligned}
q &= \left(\frac{-Z\varepsilon a\lambda\dot{G}}{c} \right) \left\{ \int_{r=0}^{r=R} (s + w\gamma\,)\lambda \left(2 - \frac{\lambda}{r} \right) \frac{e^{-\frac{\lambda}{r}}}{r^2}\, dr \right\} \\
&= -\frac{Z\varepsilon a\dot{G}}{c}(s + w\gamma_o)\left(1 \quad \frac{\lambda}{r} \right) e^{-\frac{\lambda}{R}} .
\end{aligned}
\tag{5.11}
$$

Similarly, we may denote the gravitational "charge" as

$$
\frac{M}{c} = \int_{vol} \frac{J_4}{c^2}\, dvol \quad .
$$

Then, using Eqn. (5.8), we have

$$
\begin{aligned}
\frac{M}{c} &\cong \int_{vol} \frac{-Z\, a_0\, wa\lambda \left(2 - \frac{\lambda}{r} \right)}{4\pi\mu} \left[\frac{e^{-\frac{\lambda}{r}}}{r} \right] ds \\
&\cong \frac{Z\, a_0\, wa}{\mu c} \left(1 - \frac{\lambda}{r} \right) e^{-\frac{\lambda}{r}} .
\end{aligned}
\tag{5.12}
$$

Now the electrons' force is given by

$$
F_e = q\, E_r = \left(\frac{Z\varepsilon a\dot{G}}{c} \right) \left[(s + w\gamma_0) \left(1 - \frac{\lambda}{r} \right) e^{-\frac{\lambda}{r}} \right] \left(\frac{Za\dot{G}}{c} \right) \bullet (s + w\gamma_0) \left[\frac{\left(1 - \frac{\lambda}{r} \right) e^{-\frac{\lambda}{r}}}{r^2} \right]
$$

or

$$
F_e \cong \left(\frac{Z^2 G\, a^2\, \dot{G}^2}{c^2} \right) s^2 \frac{\left(1 - \frac{\lambda}{r} \right) e^{-\frac{\lambda}{r}}}{r^2} \left(1 - \frac{\lambda}{r} \right) e^{-\frac{\lambda}{r}}
$$

since s >> $\varepsilon\gamma_0$. If we compare this with the classical expression for r >> γ, then we must have

$$
\varepsilon \left(\frac{Za\dot{G}s}{c} \right)^2 \left(1 - \frac{\lambda}{r} \right) e^{-\frac{\lambda}{r}} \cong \frac{Z^2 e^2}{4\pi\varepsilon}
$$

or

$$
a^2 s^2 \left(1 - \frac{\lambda}{r} \right) e^{-\frac{\lambda}{r}} \cong \frac{e^2 c^2}{4\pi\, \varepsilon^2\, \dot{G}^2} \quad ,
\tag{5.13}
$$

Thus,

$$as \cong \frac{\pm ec}{\sqrt{4\pi\varepsilon}\,\dot{G}\left(1-\frac{\lambda}{r}\right)^{\frac{1}{2}} e^{-\frac{\lambda}{2}r}} \quad ,$$

which gives the product of a and s in terms of the experimental quantities ε, c, e, and dG/dt.

Let us now turn to the gravitational force, which is given by

$$F_g = M\,V_r$$
$$= \left(\frac{Z\,a_0\,wa}{\mu}\right)\left(1-\frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}}(Z\,a_0\,wa)\left(1-\frac{\lambda}{r}\right)\frac{e^{-\frac{\lambda}{r}}}{r^2}$$
$$= \frac{(Z\,a_0\,wa\,)^2}{\mu}\left(1-\frac{\lambda}{r}\right)^2\frac{e^{-2\frac{\lambda}{r}}}{r^2} \quad .$$

By comparing this with the classical expression for the gravitational force we find we must have

$$(Z\,a_0\,wa\,)^2\left(1-\frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}} = \mu\,G\,M^2$$

or

$$aw = \left(\frac{m}{Z\,a_0}\right)\frac{\sqrt{G\mu}}{\left(1-\frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}}} \tag{4.14}$$

If we consider the particle to be a proton then Z = 1, and the ratio of the magnitude of the electric force to the gravitational force is given by

$$F_R = \frac{e^2}{4\pi\varepsilon G\,m^2} \quad ,$$

thus

$$G\,m^2 = \frac{e^2}{4\pi\varepsilon\,F_R}$$

and

$$(\,a_0\,wa\,)^2 = \frac{e^2\,\mu}{4\pi\varepsilon\,F_R}$$

from Eqn. (5.14). Then, we have

$$aw = \left(\frac{e}{a_0}\right)\frac{\sqrt{\mu}}{\sqrt{4\pi\varepsilon\, F_r}} = \frac{e}{\varepsilon\, a_0 c\sqrt{4\pi\, F_R}} \quad . \tag{5.15}$$

Now choose

$$a = \frac{e}{\varepsilon} \tag{5.16}$$

so that, by Eqn. (5.11),

$$b = \left(\frac{e}{\varepsilon}\right)\dot{G} \quad , \tag{5.17}$$

and, from Eqn. (5.15),

$$w = \frac{m\sqrt{\varepsilon G}}{Za_o ce\left(1 - \frac{\lambda}{r}\right)^{\frac{1}{2}} e^{-\frac{\lambda}{2r}}} \quad . \tag{5.18}$$

From Eqn. (5.13) we find

$$s = \frac{c}{4\pi\,\dot{G}\left(1 - \frac{\lambda}{r}\right)^{\frac{1}{2}} e^{-\frac{\lambda}{2r}}} \tag{5.19}$$

These values point out the extremely weak time dependence of the gravitational field and the very weak mass dependence of the electric field. This mass dependence may be better seen if we write

$$E_r = \frac{aZ\dot{G}(s + w\gamma\,)}{c}\left[\frac{\left(1 - \frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}}}{r^2}\right]$$

then substitute for the field parameters to arrive at

$$E_r = \left(\frac{Ze\dot{G}}{\varepsilon c}\right)\left[\frac{c}{4\pi\dot{G}} + \frac{\gamma c}{a_0}\sqrt{\frac{1}{4\pi\, F_r}}\right]\left[\frac{\left(1-\frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}}}{r^2}\right]$$

$$= \left(\frac{Ze}{4\pi\varepsilon}\right)\left[1 + \frac{\gamma\dot{G}4\pi}{a_0}\sqrt{\frac{\varepsilon G\, m^2}{e^2}}\right]\left[\frac{\left(1-\frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}}}{r^2}\right]$$

$$= \left(\frac{Ze}{4\pi\varepsilon}\right)\left[1 + \left(\frac{4\pi\gamma\dot{G}m}{a_0\, e}\right)\sqrt{\varepsilon G}\right]\left[\frac{\left(1-\frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}}}{r^2}\right].$$

Now let us return to the search for the charge-to-mass ratio since we have all the necessary information. The quantity we defined as the gravitational "charge" is given by Eqn. (5.12). If we divide this by c we have

$$\frac{M}{c} = \frac{Z\, a_0\, wa}{\mu c}\left(1-\frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}} \quad ,$$

but, by Eqn. (5.14), this becomes

$$\frac{M}{c} = \left(\frac{Z\, a_0}{\mu c}\right)\left(\frac{m}{Z\, a_0}\right)\sqrt{G\mu}\left(1-\frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}}$$

$$= \left(\frac{m}{\mu c}\right)\sqrt{G\mu}\left(1-\frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}}$$

$$= m\sqrt{\varepsilon G}\left(1-\frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}}$$

or, rewriting

$$\frac{M}{c} = m\sqrt{\varepsilon G}\left(1-\frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}}. \tag{4.20}$$

Thus, the charge-to-mass ratio we seek is given by

$$\beta = \left(\frac{M/c}{m}\right) = \sqrt{\varepsilon G} \quad , \tag{5.21}$$

or

$$\beta = \sqrt{\varepsilon G} = 2.4296 \text{ x } 10^{-11} \text{ coul/kg} \quad .$$

The surprising, and pleasing, thing about this result is that it is formed as the product of two known physical quantities rather than depending upon new quantities, such as a_0 and , whose values are not well known. Further, in retrospect, it appears to be the simpliest, if not the only, combination of an electromagnetic parameter and a gravitational parameter whose units are coul/kg.

It is worthwhile to point out that the dependence of the fields upon time and mass density (ie, b and w) is extremely small but is essential in establishing ß, and the inductive coupling between the electromagnetic and gravitational fields.

The charge-to-mass ratio brings up the notion that a rotating gravitational, electrically neutral body should have a magnetic moment stemming from the effective electric charge associated with the gravitational mass. Given the charge-to-mass ratio we may quickly look at its prediction for the earth's magnetic moment.

Using ß, the "effective" charge associated with a gravitational mass is given by

$$q_{eff} = \beta M \quad .$$

For the earth this effective charge would be

$$q_{eff} = 1.454 \times 10^{14}\ coul \quad .$$

Thus, if the magnetic moment of the earth is given by

$$\mu = \left(q_{eff}/2M\right)A \quad ,$$

where A is the earth's angular momentum then we have

$$\mu = \left(\frac{q_{eff}}{2M}\right)I\omega$$

$$= \frac{(1.454 \times 10^{14})(9.71 \times 10^{37})(7.29 \times 10^{-5})}{2(5.983 \times 10^{24})}$$

or

$$\mu = 8.6 \times 10^{22}\ amp - m^2$$

This predicted value of the earth's magnetic moment compares very well with the experimental value of 8.1×10^{22} amp- m^2.

5.2 Perihelion Advance

No serious suggestion that the additional vector field in the five-dimensional gauge equations of the Dynamic Theory be the gravitational field can be made without giving due consideration to the explanation of the planetary perihelion advance provided by Einstein's General Theory of Relativity. Though several attempts have been made to explain the perihelion advance by other means none has succeeded in casting much doubt on Einstein's explanation.

Let us recall some of the main features of the classical problem of planetary orbits. Kepler's first law states that a planet describes a closed elliptical orbit with the sun at a focal point. However, the presence of such small influences as other planets moving in the suns' field causes a perturbation in the motion of a given planet, and the resulting orbit is not precisely elliptic. Indeed, one may think of the actual orbit as a slightly bumpy ellipse which may precess in the plane of motion; that is, the perihelion shifts about and does not always occur at the same angular position.

The fact that the idealized classical orbit is a closed ellipse is a result peculiar to the Newtonian inverse-square law; in fact, Newton himself found that, if the force of gravity were proportional to $1/r^{(2+\delta)}$ instead of $1/r^2$, then a planetary orbit would not be closed and a perihelic shift of order δ would occur. Indeed, this result was taken to indicate that, since planetary orbits are very nearly closed, the Newtonian inverse-square law must be very accurate, as in fact it is.

Let us now ask were may there be room for differences between the predictions of classical celestal mechanics and the celestial mechanics of the General Theory of Relativity or the Dynamic Theory presented here. Since Kepler's first law is experimentally verified to be correct to a high accuracy, we might expect that non-Newtonian Theories may merely add a few bumps to the nearly elliptic orbits and contribute somewhat to perihelic motion. Since angles are much more conveniently measured in astronomy than are distances, it is natural to concentrate on perihelic motion. Conveniently enough, there is, in fact, a well-known discrepancy in classical mechanics concerning the perihelic motion of the planet Mercury. Because of Mercury's high velocity and eccentric orbit, the perihelion position can be accurately determined by observation. The difference between the classically predicted perihelic shift (due to perturbations by other planets) and the observed perihelic shift is 43 seconds of arc per century. Even though this is a very small difference, it is about a hundred times the probable observational error and represents a true discrepancy from the very precise predictions of celestial mechanics which has bothered astronomers since the middle of the last century.

The first attempt to explain this discrepancy consisted in hypothesizing the existence of a new planet, Vulcan, inside the orbit of Mercury, and much theoretical work was done to predict the position of Vulcan, using the known perturbation on Mercury's orbit. However, careful observation failed to discover the hypothetical planet, and the hypothesis was finally abandoned in 1915 when Einstein used general relativity theory to explain the observed effect.

Now let us look at what the Dynamic Theory offers as an explanation for the perihelic advance and then compare it to the predictions of the general relativity theory.

The classical equations of motion are

$$\begin{aligned} m\ddot{r} - mr\dot{\theta}^2 &= F(r) \\ mr\ddot{\theta} + 2m\dot{R}\dot{\theta} &= 0 \quad . \end{aligned} \qquad (5.22)$$

The second of Eqns. (5.22) has the solution

$$\dot{\theta} = \frac{L}{mr^2} \quad , \tag{5.23}$$

where L is the angular momentum.

Using Eqn. (5.23) the first of Eqn.s (5.22) may be written

$$M\ddot{r} = F(r) + \frac{L^2}{M\,r^3}$$

or

$$M\ddot{r} \equiv -\frac{\partial}{\partial r}\left[v'(r)\right] \quad , \tag{5.24}$$

where

$$v'(r) = v(r) + \left(\frac{L^2}{2\,Mr^2}\right) \quad . \tag{5.25}$$

We are seeking the prediction of the Dynamic Theory with respect to the perihelion advance. This may be found by comparing the frequency of small radial oscillations about steady circular motion for the effective potential given by Eqn. (5.25) for the non-singular potential of the Dynamic Theory with the frequency of revolution.

By considering the non-singular potential of the Dynamic Theory, Eqn. (5.25) becomes

$$v'(r) = \left(\frac{K}{r}\right)e^{-\frac{\lambda}{r}} + \left(\frac{L^2}{2\,Mr^2}\right) \quad , \tag{5.26}$$

with K = -GMm, where G is the gravitation constant, M is the mass of the sun, and m is the mass of the planet of interest.

Equation (5.23) gives the frequency of revolution. To determine the frequency of small radial oscillations about steady circular motion we need to evaluate the second derivative of the effective potential, v, the radius for which the first derivative is zero.

The first derivative of the effective potential is obtained by differentiating Eqn. (5.26) with respect to r. This may be found to be

$$\frac{\partial}{\partial r}\left[v'(r)\right] = \left(\frac{-K}{r^2}\right)\left(1 - \frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}} - \left(\frac{L^2}{mr^3}\right) \quad . \tag{5.27}$$

The second order derivative of the effective potential may be found to be approximately

$$\frac{\partial^2 v'(r)}{\partial r^2} \cong \left(\frac{2K}{r^3}\right)\left(1 - \frac{2\lambda}{r}\right)e^{-\frac{\lambda}{r}} + \left(\frac{3\,L^2}{mr^4}\right) \quad , \tag{5.28}$$

when terms involving λ^2/r^2 are considered negligible with respect to terms involving λ/r.

We may determine r_0 from the condition

$$\frac{\partial[v'(r)]}{\partial r} = 0 = \left(\frac{GMm}{r_0^2}\right)\left(1 - \frac{\lambda}{r_0}\right)e^{-\frac{\lambda}{r_0}} - \left(\frac{L^2}{mr_0^3}\right) \quad . \tag{5.29}$$

The radius, r_0 is the radius of near circular orbit and the effect of the exponential factor and $(1-\lambda/r)$ factor will be negligible for $\lambda << r$. Thus, we may approximate Eqn. (5.29) by

$$0 = \left(\frac{GMm}{r_0^2}\right) - \left(\frac{L^2}{mr_0^3}\right)$$

so that

$$r_0 \cong \left(\frac{L^2}{GMm^2}\right) \quad . \tag{5.30}$$

If we approximate the exponential factor in Eqn. (5.28) by its power series expansion and retaining only those terms whose dependence upon λ/r are linear or less, then Eqn. (5.28) is approximated by

$$\begin{aligned}\frac{\partial^2 v'(r)}{\partial r^2} &\cong \left(\frac{2K}{r^3}\right)\left(1 - \frac{2\lambda}{r}\right)\left(1 - \frac{\lambda}{r}\right) + \left(\frac{3L^2}{mr^4}\right) \\ &= \left(\frac{2K}{r^3}\right)\left(1 - \frac{3\lambda}{r}\right) + \left(\frac{3L^2}{mr^4}\right) \quad .\end{aligned} \tag{5.31}$$

Now the frequency of small radial oscillations about steady circular motion may be found from

$$\omega^2 \equiv \left(\frac{1}{m}\right)\left[\frac{\partial^2 v'(r)}{\partial r^2}\right] r = r_0 \quad ,$$

Thus, we have

$$\begin{aligned}\omega^2 &\cong \left(\frac{1}{m}\right)\left\{-2GMm\left(\frac{GMm^2}{L^2}\right)^3\left[1 - \left(\frac{3\lambda GM\,m^2}{L^2}\right)\right] + \left(\frac{3L^2}{m}\right)\left(\frac{GMm^2}{L^2}\right)^4\right\} \\ &= \left(\frac{-2G^4M^4m^6}{L^6}\right)\left[1 - \left(\frac{3\lambda GM\,m^2}{L^2}\right)\right] + \frac{3G^4M^4m^6}{L^6} \\ &= \left(\frac{G^4M^4m^6}{L^6}\right)\left[1 + \left(\frac{6\lambda GM\,m^2}{L^2}\right)\right] \quad .\end{aligned} \tag{5.32}$$

An approximation for the frequency of small radial oscillations about steady circular motion may now be made by taking the square root of Eqn.

(5.32) and considering the second terms of the second factor as small compared to one. Thus, we have

$$\omega \cong \left(\frac{G^2 M^2 m^3}{L^3}\right)\left[1+\left(\frac{3\lambda GM\, m^2}{L^2}\right)\right] \quad . \tag{5.33}$$

The perihelion advance per revolution may now be found as the difference between Eqns. (5.33) and Eqn. (5.23) evaluated at r_0, divided by the orbital frequency, or

$$\delta\theta = 2\pi \left(\frac{\omega - \dot{\theta}}{\dot{\theta}}\right)$$

$$= 2\pi \left\{\left(\frac{G^2 M^2 m^3}{L^3}\right)\left[1+\left(\frac{3\lambda GM\, m^2}{L^2}\right)\right] - \frac{G^2 M^2 m^3}{L^3}\right\}\left(\frac{L^3}{G^2 M^2 m^3}\right)$$

so that

$$\delta\theta \cong 2\pi \left(\frac{3\lambda\, GMm^2}{L^2}\right) \quad .$$

The perihelion advance predicted by Einstein's General Theory of Relativity is given by

$$\delta\,\theta_{GiR} = 2\pi \left(\frac{3\, G^2 M^2 m^2}{c^2 L^2}\right) \quad .$$

If λ were to be such as to provide an identical prediction as the General Theory then λ would have to satisfy

$$\lambda = \frac{GM}{c^2}$$

For G = 6.7 x 10^{-8} $gr^{-1}cm^3/sec^2$, M = 1.98 x 10^{33}gr, and c = 3 x 10^{10} cm/sec,

$$\lambda = \frac{(6.7 \text{ x } 10^{-8})(1.98 \text{ x } 10^{33})}{(3 \text{ x } 10^{10})^2}\, cm = 1.47 \text{ x } 10^{15}\, cm$$

or

$$\lambda = 1.47 \text{ x } 10^3\, m \quad .$$

This is an extremely small value compared to the radius of the sun but is sufficient within the Dynamic Theory to provide the same prediction of perihelion advance as the General Theory of Relativity.

5.3 Redshifts

Einstein's General Theory of Relativity predicted the advance of the perihelion of planetary orbits by using the full effect of the geometrical equations. We saw in the previous section that the Dynamic Theory predicts a similar planet orbit perihelion advance. Another of the predictions of Einstein's Theory concerns the redshifts associated with light received from distant light emitting objects or when light travels through a changing gravitational field.

The Dynamic Theory should also predict frequency shifts that are experimentally measurable. If it does not then it doesn't have the same strength of predictability as Einstein's General Theory.

There are two types of redshifts resulting from the theoretical approach of the Dynamic Theory. First, there is an expansion red shift due to the increasing "entropy" of the universe. Secondly, there is a frequency shift caused by a difference in the gravitational strength between the point of emission of the light and the point of its reception. Both of these types of frequency shifts are the result of a difference in the effective unit of action at the emission point and the reception point.

Both of the above types of frequency shifts may be referred to as of geometrical in origin in that they both come from the gauge function. However, each originates from a different variable change in the gauge function. For instance the expansion shift involves considering the universe as an isolated system resulting in the entropy principle requiring small frequency shifts toward the red. This comes from the gauge function being dependant upon time. The second type of frequency shift comes from the gauge functions dependence upon space and mass.

We may first consider these types of frequency shifts to be independent and look at each in turn. Then we shall consider them together. First, we will need to consider the local systems where a photon is first emitted and then where it is received. In both systems the energy of a photon is given by hν, where ν is the frequency and h is the effective unit of action. The effective unit of action is the product of the gauge function and Planck's constant if a locally flat metric is considered.

At the heart of both types of frequency shifts is the gauge function which has previously been given as

$$ln\, f^{\frac{1}{2}} = f_r f_t f_\theta f_\phi f_\gamma \tag{5.34}$$

and found to be

$$ln\, f^{\frac{1}{2}} = f_t f_\gamma \left(\frac{e^{-\frac{\lambda}{r}}}{r} \right) ,$$

where f_t and f_γ indicates functions of time and mass density.

We need to determine more about the gauge function than we previously have. The square of the arc lengths differ by the multiplicative gauge function as

$$(dq^0)^2 = f(d\sigma)^2$$

or

$$(dq^0)^2 = \exp\left[2 f_\varepsilon f_\gamma\left(\frac{e^{-\frac{\lambda}{r}}}{r}\right)\right](d\sigma)^2 \quad .$$

From this we see that the differential change in entropy is given by

$$dq^0 = \exp\left[f_t f_\gamma\left(\frac{e^{-\frac{\lambda}{r}}}{r}\right)\right]d\sigma \quad . \qquad (5.35)$$

Recalling that there can be no decrease in the entropy for an isolated system we must then consider the possible effect of the entropy principle upon the universe as an isolated system. We can see from this line of thinking that f_t in Eqn. (5.35) is the one to focus on for the moment. The simplest function is of course the linear function and this linear dependence appeared previously in section 5.1.

Suppose then, we consider the effective unit of action for an isolated universe at some time which we will set at t = 0. We find

$$\overline{h}_0^{\prime} = \overline{h} f_0 \quad ,$$

which can be written

$$\overline{h}_0^{\prime} = \overline{h}\exp[0] \qquad (5.36)$$

at t = 0. Here the value of the effective unit of action corresponds identically with Planck's constant, h.

At some later time t = T the effective unit of action would be given by

$$\overline{h}_T^{\prime} = \overline{H}\exp[AT] \quad ,$$

where A is a constant. Let us further consider a change of variables using the distance light will travel in free space instead of the time, T. Since T = L/C, L is the distance variable we seek. We now have

$$\overline{h}_T^{\prime} = \overline{H}\exp\left[\frac{AL}{c}\right] \quad . \qquad (5.37)$$

If a photon were emitted at t = 0 it would have been emitted with an effective unit of action given by Eqn. (5.36). If that photon is received at the later time, T, at a distance of L, then the universe's effective unit of action would be given by Eqn. (5.37) at reception. If the energy of the photon when emitted is given by $\bar{h}_0^{1}\nu_e$ 85, then no loss of energy by the photon until reception would require that

$$\bar{h}_0^{1}\nu_e = \bar{h}_T^{1}\nu_r \tag{5.38}$$

Substituting from Eqn. (5.36) and Eqn. (5.37) into Eqn. (5.38) we find

$$\nu_e = \nu_r \exp\left[\frac{AL}{c}\right] \ . \tag{5.39}$$

The frequency shift would be given by

$$\frac{\Delta\nu}{\nu_e} = \frac{\nu_r - \nu_e}{\nu_e} = \frac{\left(1 - e^{\frac{AL}{c}}\right)}{e^{\frac{AL}{c}}}$$

or

$$\frac{\Delta\nu}{\nu_e} = e^{-\frac{AL}{c}} - 1 \ . \tag{5.40}$$

The question arises whether or not the frequency shift given by Eqn. (5.40) is red or blue? From Eqn. (5.40) it may be seen that the frequency shift is negative if A > 0, thus the shift is red or blue as A is positive or negative. Going back to Eqn. (5.35) and using the gauge function of Eqn. (5.37) we see that

$$dq^0 = \left(e^{\frac{AL}{c}}\right) d\sigma$$

This indicates that a given element of arc length, $d\sigma$, yields a larger change in entropy, dq^o, at the time T = L/C than before at time t = 0. Thus, the entropy change is increasing and our universe is expanding.

The expansion red shift given by Eqn. (5.40) may also be expressed in terms of wavelength as

$$\frac{\Delta\lambda}{\lambda_e} = \frac{\nu_e}{\nu_r} - 1 = e^{\frac{AL}{c}} - 1 \ . \tag{5.41}$$

Equation (5.41) may be expanded as

$$\frac{\Delta\lambda}{\lambda_e} = \left(\frac{AL}{c}\right) + \frac{1}{2!}\left(\frac{AL}{c}\right)^2 + \frac{1}{3!}\left(\frac{AL}{c}\right)^3 + \ldots \tag{5.42}$$

which may be approximated by

$$\frac{\Delta\lambda}{\lambda_e} \cong \frac{AL}{c} \tag{5.43}$$

when AL << c.

Experimentally it has been determined that

$$\frac{\Delta\lambda}{\lambda_e} \cong \frac{HL}{c} \quad ,$$

where H is the Hubble constant which is given by

$$H^{-1} = (5.6 +\_\ 0.6)\ x\ 10^{17}\ \text{sec}.$$

Then we find the predicted frequency shift given by Eqn. (5.42) is seen in nature when the constant, A, is taken as the Hubble constant.

From Eqn. (5.41) we see that experimentally found red shifts can be used as astronomical markers from the expression

$$L = \left(\frac{c}{H}\right) \ln\left[1 + \left(\frac{\Delta\lambda}{\lambda_e}\right) \exp\right] \quad . \tag{5.44}$$

For small experimental red shifts, compared to one, the astronomical distances L, given by Eqn. (5.44), are not significantly different from the linear markers given by the approximation of Eqn. (5.43). However, as the red shift begins approaching unity the difference becomes significant.

Turning to the second type of frequency shift, and returning to Eqn. (5.34) to again write

$$\ln f^{\frac{1}{2}} = f_\varepsilon f_\gamma \left(\frac{e^{-\frac{\lambda}{r}}}{r}\right) \quad . \tag{5.45}$$

The effective unit of action is expressed by

$$\bar{h}^{I} \int_{jk} = \bar{h} f\, \hat{g}^{kl} \left[\int_{ie} + \begin{Bmatrix} j \\ s\, l \end{Bmatrix} x^s\right] \quad . \tag{5.46}$$

But when in a locally Euclidean geometry the $g^{kl} = \delta^{kl}$ then Eqn (5.46) becomes

$$\bar{h}^{I} = \bar{h} f = \bar{h} \exp\left[2 f_t f_\gamma \left(\frac{e^{-\frac{\lambda}{r}}}{r}\right)\right] \quad . \tag{5.47}$$

From the expressions for the gauge potentials

$$\phi_j = \frac{1}{2}\frac{\partial(lnf)}{\partial x^j} = \frac{\partial(ln\, f^{\frac{1}{2}})}{\partial x^3} \tag{5.48}$$

and

$$F_{ij} = \phi_{i,j} - \phi_{j,i} \tag{5.49}$$

Using Eqn. (5.49), F_{14} gives the radial component of the gravitational field. From Eqn. (5.48) we find

$$\phi_1 = \frac{\partial(ln\, f^{\frac{1}{2}})}{\partial r} = -f_t f_\gamma \left(\frac{e^{-\frac{\lambda}{r}}}{r^2}\right)\left(1 - \frac{\lambda}{r}\right)$$

and

$$F_{14} = -a_o f_t \left(\frac{df_\gamma}{d\gamma}\right)\left(1 - \frac{\lambda}{r}\right)\left(\frac{e^{-\frac{\lambda}{r}}}{r}\right) \quad . \tag{5.50}$$

The expression for the gravitational field given in Eqn. (5.50) is in terms of a field density. By integration over the volume occupied by the gravitational mass density we have the gravitational field

$$V_r = -a_o f_t \left(\frac{df_M}{dM}\right)\left(1 - \frac{\lambda}{r}\right)\left(\frac{e^{-\frac{\lambda}{r}}}{r}\right) \quad , \tag{5.51}$$

where M is the total gravitating mass.

For any weak time variation in the field we can ignore the time dependence. Thus, we have

$$V_r = -a_o a \left(\frac{df_M}{dM}\right)\left(1 - \frac{\lambda}{r}\right)\left(\frac{e^{-\frac{\lambda}{r}}}{r}\right) \tag{5.52}$$

to be compared with the experimental field

$$V_r \exp = \frac{GM}{r^2} \quad . \tag{5.53}$$

Certainly for $r >> \lambda$ V_r in Eqn. (5.52) is approximated by the experimental expression of Eqn. (5.53) if A = -G and

$$\frac{df_M}{dM} = M \quad .$$

From Eqn. (5.54) we find that

$$f_M = \frac{1}{2} M^2 + constant.$$

The conversion to a field density may be done by dividing by the mass, M, so that the gauge function in Eqn (5.47) becomes

$$\bar{h}^1 = \bar{H} \exp\left[\frac{-2G\left(\frac{1}{2} M^2 + constant\right)}{M c^2} \left(\frac{e^{-\frac{\lambda}{r}}}{r} \right) \right]$$

or

$$\bar{h}^1 = \bar{h} \exp\left[\left(\frac{-GM}{c^2} \right) \left(\frac{e^{-\frac{\lambda}{r}}}{r} \right) \right] , \tag{5.55}$$

where c^2 has been used to obtain a unitless quantity, which must be the case for f.

Now, using the unit of action given by Eqn. (5.55), suppose a photon is emitted from one body with a gravitational field

$$\left(\frac{GM_1}{c^2 R_1} \right) e^{-\frac{\lambda_1}{R_1}} ,$$

and is received in another gravitational field

$$\left(\frac{GM_2}{c^2 R_2} \right) e^{-\frac{\lambda_2}{R_2}}$$

the conservation of photon energy would then require

$$\bar{h}_{e'} \nu_e = \bar{h}_{r'} \nu_r$$

or

$$\nu_e \exp\left[\left(\frac{-GM_1}{c^2 R_1} \right) \left(e^{-\frac{\lambda_1}{R_1}} \right) \right] = \nu_r \exp\left[\left(\frac{-GM_2}{c^2 R_2} \right) \left(e^{-\frac{\lambda_2}{R_2}} \right) \right]$$

so that

$$\nu_r = \nu_e \exp\left[\left(\frac{-GM_1}{c^2 R_1} \right) e^{-\frac{\lambda_1}{R_1}} + \left(\frac{GM_2}{c^2 R_2} \right) e^{-\frac{\lambda_2}{R_2}} \right] . \tag{5.56}$$

The frequency shift would then be

$$\frac{\Delta\nu}{\nu_e} = \frac{\nu_r - \nu_e}{\nu_e} = \exp\left[\left(\frac{GM_2}{c^2 R_2}\right)e^{-\frac{\lambda_2}{R_2}} - \left(\frac{GM_1}{c^2 R_1}\right)e^{-\frac{\lambda_1}{R_1}}\right] \tag{5.57}$$

For $R_2 >> \lambda_2$ and $R_1 >> \lambda_1$, then Eqn. (5.57) may be approximated by

$$\frac{\Delta\nu}{\nu_e} \cong e\left\{\left(\frac{G}{c^2}\right)\right\} - 1 \quad . \tag{5.58}$$

The approximation in Eqn. (5.58) shows that if $M_1/R_1 > M_2/R_2$ then this frequency shift given by Eqn. (5.57) is negative, or towards the red end of the spectrum.

We can make the further simplification of assuming that both

$$\frac{GM_1}{c^2 R_1} << 1 \quad and \quad \frac{GM_2}{c^2 R_2} << 1 \quad ,$$

then we would have the approximation that

$$\frac{\Delta\nu}{\nu_e} - \frac{GM_2}{c^2 R_2} - \frac{GM_1}{c^2 R_1} \quad . \tag{5.59}$$

In terms of wavelengths we have

$$\begin{aligned} \frac{\Delta\lambda}{\lambda_e} &= \frac{\nu_e}{\nu_r} - 1 \\ &= \exp\left[\left(\frac{GM_1}{c^2 R_1}\right)e^{-\frac{\lambda_1}{R_1}} - \left(\frac{GM_2}{c^2 R_2}\right)e^{-\frac{\lambda_2}{R_2}}\right] \end{aligned} \tag{5.60}$$

with the above assumptions Eqn. (5.60) is approximated by

$$\frac{\Delta\lambda}{\lambda_e} - \frac{GM_1}{c^2 R_1} - \frac{GM_2}{c^2 R_2} \quad . \tag{5.61}$$

Suppose we look at this red shift for a photon emitted from the surface of the sun and received at the earth's surface. The needed numbers are:

$G = 6.67x10^{-11}$nt-m^2/kg^2 $M_{sun} = 329,390(5.983x10^{24}$kg)
$M_{earth} = 5.983x10^{24}$kg $R_{sun} = 6.953x10^8$m
$R_{earth} = 6.371x10^6$ $c = 3x10^8$ m/sec

Thus, from Eqn. (5.61),

$$\frac{\Delta\lambda}{\lambda_e} \cong 2.1 \, x \, 10^{-6} \quad ,$$

or 0.63 in km/sec. This is the same as predicted by Einstein's General Theory of Relativity.

For a terrestrial test of the red shift the prediction would be

$$\frac{\Delta\lambda}{\lambda_e} - \left(\frac{G}{c^2}\right)\left[\left(\frac{M}{R}\right) - \left(\frac{M}{R+\Delta R}\right)\right] = \left(\frac{GM}{c^2 R^2}\right)\Delta R \quad .$$

If ΔR = 72 ft = 21.95m, then $\Delta\lambda/\lambda e \approx 2.4 \times 10^{-15}$.

Since the approximation given by Eqn. (5.59) may be expressed as

$$\frac{\Delta\nu}{\nu_e} - \frac{\Delta d}{c^2} \quad ,$$

where $\Delta\varphi = G[(M_e/R_e)-(M_r/R_e)]$, then it may be seen that the red shift given by Eqns. (5.57) and (5.60) produce the red shifts predicted by Einstein's General Theory of Relativity if $R_1 >> \lambda_1$, $R_2 >> \lambda_2$, $GM_1 << c^2R_1$, and $GM_2 << c^2R_2$. However, if these conditions of approximation are not met then one must resort back to Eqn.'s (5.57) and (5.60) for the predicted red shifts.

Suppose one considered a photon which may have been emitted on a dense star such that GM_1/C^2R_1 is too large to allow a simplification of the exponential expression. If this photon were received on earth then, by Eqn. (5.60), we would have

$$\left(\frac{\Delta\lambda}{\lambda_e}\right)_{\exp} \cong \exp\left[\left(\frac{G}{c^2}\right)\left(\frac{M_e}{R_e} - \frac{M\,earth}{R\,earth}\right)\right] - 1 \quad .$$

Because of the small value of $(GM_{earth})/(C^2R_{earth})$ 7 x 10^{-10}, then the approximation becomes

$$\left(\frac{\Delta\lambda}{\lambda^e}\right)_{\exp} - e^{\left(\frac{GM_2}{c^2 r_2}\right)} - 1 \quad , \tag{5.62}$$

where GM/c^2R is the gravitational field at photon emission.

From Eqn. (5.62) we may learn something of a stars' density by the red shift in the light received from it. For instance, Eqn. (5.62) has

$$\frac{GM}{c^2 R} - ln\left[1 + \left(\frac{\Delta\lambda}{\lambda_e}\right)_{\exp}\right] \quad . \tag{5.63}$$

Notice that even the large red shifts displayed by quasars are allowed by Eqn. (5.63) without requiring them to be at the far reaches of our universe.

We have looked at the predicted frequency shifts as if they were independent phenomena. In the sense that they stem from the same gauge function, then both types of redshifts may be present in any light received. Thus, we should look at the expression for the entire red shift.

Suppose we let

$$ln\, f^{\frac{1}{2}} = f_r f_t f_\gamma = (s + w\gamma + z\gamma^2)(a+bt)\left(\frac{k}{r}\right)e^{-\frac{\lambda}{R}}$$

where s, w, z, a, b, and k are to be evaluated. The effective unit of action becomes

$$\bar{h}_0^1 = \bar{h}\exp\left[\left(\frac{k}{r}\right)e^{-\frac{\lambda}{r}}(s + w\gamma + z\gamma^2)(a+bt)\right] , \qquad (5.64)$$

for t = 0. From the above, let us take s = w = 0, then Eqn. (5.65) becomes

$$\bar{h}_e^1 = \bar{h}\exp\left[\left[\frac{kz(a+bt_e)M_e}{R_e}\right]e^{-\frac{\lambda_e}{R_e}}\right] , \qquad (5.65)$$

when integrated over the entire gravitating mass as before and the subscript, e, refers to the unit of action at the place and time of emission of the photon. A similar expression is found at the place and time of reception, or

$$\bar{h}_r^1 = \bar{h}\exp\left\{\left[\frac{kz(a+bt_r)M_r}{R_r}\right]e^{-\frac{\lambda_r}{R_r}}\right\} . \qquad (5.66)$$

Equations (5.65) and (5.66) may be used to express the frequency shift required to conserve photon energy, since

$$\bar{h}_e^1 \nu_e = \bar{h}_r^1 \nu_r .$$

Thus, we have

$$\nu_r = \nu_e \exp\left\{\left[\frac{zkM_e(a+bt_e)}{R_e}\right]e^{-\frac{\lambda_e}{R_e}} - \left[\frac{zkM_r(a+bt_r)}{R_r}\right]e^{-\frac{\lambda_r}{R_r}}\right\} ,$$

then

$$\frac{\Delta\nu}{\nu_e} = \exp\left\{ zk\left[\frac{M_e(a+bt_e)}{R_e}\right]e^{-\frac{\lambda_e}{R_e}} - \frac{M_r(a+bt_r)}{R_r}e^{-\frac{\lambda_r}{R_r}}\right\}$$

Similarly, the expression for the wavelength shift becomes

$$\frac{\Delta\lambda}{\lambda_e} = \exp\left\{ zk\left[\frac{M_r(a+bt_r)}{R_r}e^{-\frac{\lambda_r}{R_r}} - \frac{M_e(a+bt_e)}{R_e}\right]\right\} - 1 \quad . \tag{5.67}$$

If we write Eqn. (5.67) as a power series and make the approximation of keeping only the first term, we find

$$\frac{\Delta\lambda}{\lambda_e} \cong zk\left[\frac{M_r(a+bt_r)e^{-\frac{\lambda_r}{R_r}}}{R_r} - \frac{M_e(a)e^{-\frac{\lambda_e}{R_e}}}{R_e}\right] \quad , \tag{5.68}$$

where we've also let $t_e = 0$. By letting $t_r = L/c$ Eqn. (5.68) becomes

$$\frac{\Delta\lambda}{\lambda_e} \cong zk\left[\frac{M_r\left(a+\frac{bL}{c}\right)e^{-\frac{\lambda_r}{R_r}}}{R_r} - \frac{M_e a e^{-\frac{\lambda_e}{R_e}}}{R_e}\right] \quad . \tag{5.69}$$

We want to evaluate the unknowns a, k, z, and b in terms of the previously determined quantities such as G, c, and H. Therefore, suppose that the gravitational field at the time of emission of the photon is the same as here on earth at its reception, then we find Eqn. (5.69) becomes

$$\frac{\Delta\lambda}{\lambda_e} \cong \frac{zkbLM_{earth}}{CR_{earth}} \quad . \tag{5.70}$$

Experimentally, we have found the expansion red shift is given by

$$\left(\frac{\Delta\lambda}{\lambda_e}\right)_{\exp} = \frac{HL}{c} \quad ,$$

thus, we should have

$$H = \frac{zkbM_{earth}}{R_{earth}} \quad . \tag{5.71}$$

On the other hand if the time between photon emission and reception is sufficiently close then our approximation to Eqn. (5.69) can be written as

$$\frac{\Delta\lambda}{\lambda_e} - zka\left[\frac{M_r}{R_r} - \frac{M_e}{R_e}\right] , \tag{5.72}$$

where $R_r >> \lambda_r$ and $R_e >> \lambda_e$. Thus, in order to compare with experiment, we set

$$zka = -\frac{G}{c^2} . \tag{5.73}$$

If we now substitute Eqns. (5.71) and (5.73) into Eqn. (5.68), we have

$$\frac{\Delta\lambda}{\lambda_e} \cong \left(-\frac{G}{c^2}\right)\left[\frac{M_r e^{-\frac{\lambda_r}{R_r}}}{R_r} - \frac{M_e e^{-\frac{\lambda_e}{R_e}}}{R_e}\right] + \left(\frac{HL}{c}\right)\left(\frac{M_r}{R_r}\right)\left(\frac{R}{M}\right) e^{-\frac{\lambda_r}{R_r}} , \tag{5.74}$$

where R/M is the radius and mass of the earth as in Eqn. (5.71). While the full expression, Eqn. (5.67) becomes

$$\frac{\Delta\lambda}{\lambda_e} = \exp\left\{\left(\frac{-G}{c^2}\right)\left[\frac{M_r e^{-\frac{\lambda_r}{R_r}}}{R_r} - \frac{M_e e^{-\frac{\lambda_e}{R_e}}}{R_e}\right] + \left(\frac{HL}{c}\right)\left(\frac{M_r}{R_r}\right)\left(\frac{R}{M}\right) e^{-\frac{\lambda_r}{R_R}}\right\} - 1. \tag{5.75}$$

From Eqn. (5.75) it may be seen that, if the receiving location is the earth, then the time dependence in Eqn. (5.75) is given by

$$\frac{HL}{c} = Ht$$

so that b, of Eqn. (5.67) is given by -H. Thus, the gauge function dependence upon time is also given by b= -H. Recall that a time dependence of the gravitational field has been reported by T. Van Flandern with a reported value of $b = -1.9 \times 10^{-18} sec^{-1}$. The measured Hubble constant is $H^{-1} = (5.6 \pm 0.6) \times 10^{17}$ sec so that 2.00×10^{-18} sec $\geq H \geq 1.61 \times 10^{-18}$ sec.

Thus, we see that the Dynamic Theory predicts that the expansion redshift and a time decrease in the gravitational field strength are both the result of the time variation of the gauge function. Further, it seems amazing that two such different and difficult type measurements have such a good agreement!

Returning to the wavelength shift given by Eqn. (5.75), it may be seen that the contribution of expansion, or gravitational potential, to the total red shift is contained within this equation. Equation (5.75) has three unknowns: the astronomical distance L, the mass of the emitting star (or object), M_e and the size of the emitting star, R_e. Given only two pieces of experimental data such as the redshift and the apparent luminosity, we can determine the astronomical distance, L, and the gravitational density, M_e/R_e by assuming a mechanism for the light production (ie, sun-like). Given another data point such as light fluctuation periodicity then a limiting size might be obtained.

5.4 "Fifth" Force

Is a "fifth" force really necessary? Obviously, a new force is much more exciting than finding an explanation for the measured effect within another force description. A new force may be more tenable because it may appear not to compete with existing forces. A correction to existing forces may lack excitement and must certainly be shown to be compatible with existing forces where they are measured to great accuracy. A correction to an existing force is usually difficult to find and may go against the preference of many. But to assume there is an additional force is to assume its independence and would then necessitate yet another force to be "unified".

Accurate measurements show that the gravitational force of the Earth differs from Newton's Law at close range. More specifically, the difference in the Earth's gravitational field over a difference of height in a deep well is not the same as predicted by Newton's Law. This leads to a simple choice; either Newton's Law of gravity needs to have a correction or an independent fifth force is needed to explain the difference. In the past when we found that the proton-proton scattering data differed from the coulombic predictions we opted for an independent force and have had the fun of searching for a method of unifying electromagnetism and the strong force every since. We could make that same choice here, or we could investigate the difference in the prediction of the non-singular potential and Newton's law of gravity. To do this we need to look at the gravitational attraction on a mass in a well deep down from the surface of the Earth. Freshman texts on physics typically show how to calculate the gravitational influence of a thin spherical shell on a mass both inside and outside of the shell. This is the procedure we need here because a mass in a deep well feel the influence of both the mass of the Earth interior to it and in the shell of the Earth exterior to it where the shell thickness is the depth of the well. If we recall the procedure for this using the Newtonian potential, then we remember that, for the 1/r potential, all of the mass interior to the test mass attracts it as if the mass were located at the center of the Earth. On the other hand we recall that there is no gravitational influence due to the mass in the outer shell which is exterior to the test mass in the deep well.

For a potential which differs from the 1/r Newtonian potential these conclusions may not be true. Indeed, ones first suspicion is that they are not the correct conclusions. What we now need to do is to calculate the influence on a test mass both outside and inside of gravitating mass.

First, suppose we calculate the gravitational influence on a mass exterior to a thin spherical shell. Using the neo-Newtonian potential we find the integral to be

$$F = \int_{x=R-r}^{R+r} \left[\frac{-\pi G\rho rtm}{R^2}\right]\left[\frac{R^2 - r^2}{x^2} + 1\right]\left(1 - \frac{\lambda}{x}\right)e^{-\frac{\lambda}{x}}dx$$

$$F = \int_{x=R-r}^{R+r} \frac{d}{dx}\left[\frac{f(x)}{x}e^{-\frac{\lambda}{x}}\right]dx$$

Figure 12. Neo-Newtonian Potential.

Figure 13. Neo-Newtonian Force.

Figure 14. Gravitational attraction of a section dS of a spherical shell of matter on m.

where by making use of the integral tables and a lot of algebra we arrive at the solution

F is an improper integral for R=r. This means that terms in the series have denominators which tend to zero as R tends to r. We must show that the series converges because this is the case when our test mass is at the bottom of a deep well. It would then be at the outer surface of the inner mass. It is easy to see that as λ tends to zero the solution tends to the classical solution.

If we now consider our test mass to be inside a thin shell and look at the force.

$$F = \left(\frac{-\pi G\rho rtm}{R^2}\right)\int_{x=-(R-r)}^{R+r} \frac{d}{dx}\left[\frac{f(x)}{x}e^{-\frac{\lambda}{x}}\right]dx$$

$$= \left(\frac{-\pi G\rho rm}{R^2}\right)\left\{2r\left[e^{-\frac{\lambda}{R+r}} - e^{-\frac{\lambda}{-(R-r)}}\right] + 2\lambda \log\left[\frac{r-R}{R+r}\right] + 2\lambda\sum_{N=1}^{\infty}\frac{(-\lambda)^N}{N.N!}\left[\frac{1}{(R+r)^N} - \frac{1}{(r-R)^N}\right]\right\}$$

as $\lambda \to 0$, $F \to 0$ which is the classical result.

Suppose now we look at a couple of approximations which may give some insight into the influence of the exponential term in the potential. First, let us consider a mass outside another mass for which $R-r>>\lambda$, then we would have the approximation

$$F \cong \left(\frac{-GMm}{R^2}\right)\left[1 - \lambda\left(\frac{2R-r}{R^2-r^2}\right)\right] ;if \;\; R-r>>\lambda.$$

This result shows that the force of attraction on a test mass outside another mass is reduced by the second term in the square brackets. This is, of course what we should have expected from a potential which deviates from the Newtonian potential by turning around and going back to zero. The first deviation from Newtonian-like character would be to become weaker.

The other approximation to consider is that for the expression for the force an the test mass inside the shell. For this we find that inside the shell if r-R>>λ, then

$$F \cong = \left(\frac{-GMm}{R^2} \right) (-\lambda) \left[\frac{r^2 + rR - R^2}{r(r^2 - R^2)} \right] ;if \;\; r - R >> \lambda.$$

This is a force away from the center of the shell and toward the inside of the shell. The Big Question is: What is the force when the test mass is on the immediate exterior, or interior, of a shell? That is, do we have convergence of the infinite series in the solutions for both the inside and outside forces on the test mass.

To address this consider the absolute value of the ratio of the n+1 and the n^{th} terms in the force for a mass outside of a shell of finite thickness t, then from our previous results we found that

$$F = \left[\frac{-\pi G \rho t r m}{R^2} \right] \left\{ 2r \left[e^{-\frac{\lambda}{R+r}} + e^{-\frac{\lambda}{R-r}} \right] + 2\lambda \log \left(\frac{R-r}{R+r} \right) \right.$$
$$\left. + 2\lambda \sum_{N=1}^{\infty} \frac{(-\lambda)^N}{N.N!} \left[\frac{1}{(R+r)^N} - \frac{1}{(R+r)^N} \right] \right.$$

The question arises whether or not F is finite for t > 0? If R=r+t/2 then R+r=2r+ t/2 and R-r = t/2, or r = R - t/2 ∴ R+r = 2R - t/2 ; R-r = t/2

$$F = \left[\frac{-\pi G \rho \, t r^2 m}{R^2} \right] \left\{ 2 \left[e^{-\frac{\lambda}{2R - \frac{t}{2}}} + e^{-\frac{\lambda}{\frac{t}{2}}} \right] + \left(\frac{2\lambda}{R - \frac{t}{2}} \right) \log \left[\frac{\frac{t}{2}}{2R - \frac{t}{2}} \right] \right.$$
$$\left. + \frac{2\lambda}{\left(R - \frac{t}{2} \right)} \sum_{N=1}^{\infty} \frac{(-\lambda)^N}{N.N!} \left[\frac{1}{\left(2R - \frac{t}{2} \right)^N} - \frac{1}{\left(\frac{t}{2} \right)^N} \right] \right\}$$

if we have 2R>>t/2 and 2R>>λ then we may make the following approximation,

$$F \cong \left(\frac{-GMm}{R^2}\right)\left\{\frac{1}{2}\left[e^{-\frac{\lambda}{2R}} + e^{-\frac{\lambda}{\frac{t}{2}}}\right] + \left(\frac{\lambda}{2R}\right)\log\left(\frac{\frac{t}{2}}{2R}\right)\right.$$

$$\left. + \left(\frac{\lambda}{2R}\right)\sum_{N=1}^{\infty}\frac{(-\lambda)^N}{N.N!}\left[\frac{1}{(2R)^N} - \frac{2^N}{(t)^N}\right]\right\}$$

Now look at the ratio of two successive terms in the series

$$\left|\frac{a_{n+1}}{a_N}\right| = \frac{\left|\frac{(-\lambda)^{N+1}}{(N+1)(N+1)!}\left[\frac{1}{(2R)^{N+1}} - \frac{2^{N+1}}{t^{N+1}}\right]\right|}{\left|\frac{(-\lambda)^N}{N.N!}\right|\left|\left[\frac{1}{(2R)^N} - \frac{2^N}{t^N}\right]\right|}$$

$$= \left[\frac{\lambda}{(N+1)^2 tR}\right]\left[\frac{(2R)^{N+1} - \left(\frac{t}{2}\right)^{N+1}}{(2R)^N - \left(\frac{t}{2}\right)^N}\right]$$

But R>>t/2 so that

$$\lim_{N\to\infty}\left|\frac{a_{N+1}}{a_N}\right| \cong \lim_{N\to\infty}\left[\frac{2\lambda}{(N+1)^2 t}\right]$$

Therefore, the series converges absolutely!

Now how about an expression for the force for a large number of shells of thickness t sufficient to make up a sphere? If there are M shells making up the sphere then the thickness t = R_e/M. While if we want to monitor the force on a test mass at the surface of the sphere then the limits on x in the integration varies with each shell.

For instance for the outer shell the limits on x would be $R - r = \frac{t}{2} \le x \le R + r = 2R - \frac{t}{2}$ but $R = R_e$ and $t = \frac{R_e}{M}$ 154 or the limits on x would be

$$\frac{R_e}{2M} \le x \le 2R_e - \frac{R_e}{2M} = R_e\left[2 - \frac{1}{2M}\right]$$

For the next shell $R - r = \frac{3}{2}t \le x \le R + r = 2R - \frac{3t}{2}$ 156 or

$$\frac{3}{2}\frac{R_e}{M} \leq x \leq 2\,R_e - \frac{3}{2}\frac{R_e}{M} = R_e\left[2 - \frac{3}{2M}\right]$$

Thus, for the N[th] shell we have limits given by

$$\left(\frac{2N-1}{2}\right)\left(\frac{R_e}{N}\right) \leq x \leq R_e\left[2 - \frac{(2N-1)}{2N}\right]$$

Then we have

$$F(N) = \left[\frac{-GM(N)m}{R_e^2}\right]\left\{\frac{1}{2}\left[e^{-\frac{\lambda}{R_e\left[2-\frac{(2N-1)}{N}\right]}} + e^{-\frac{\lambda}{R_e\left(\frac{2N-1}{2N}\right)}}\right]\right.$$

$$+\frac{\lambda}{R_e\left[2-\frac{(2N-1)}{N}\right]}\left\{\log\left[\frac{2N-1}{4N-(2N-1)}\right]\right.$$

$$\left.\left.+\sum_{N=1}^{\infty}\frac{(-\lambda)^N}{N-N!\,R_e^N}\left[\frac{(2N)^N}{[4N-2N+1]^N} - \frac{(2N)^N}{(2N-1)^N}\right]\right\}\right\}$$

Now, looking at

$$\exp_1 = \frac{-\lambda}{R_e\left[2-\frac{(2N-1)}{N}\right]} \quad if\;\; N=0\; \_\; \exp_1 = \frac{-\lambda}{R_e\left[2-\frac{1}{N}\right]}$$

$$if\;\; N=N\; \_\; \exp_1 = \frac{-\lambda}{R_e\left[\frac{1}{N}\right]}$$

on the other hand for

$$\exp_2 = \frac{-\lambda}{\frac{R_e}{N}\left(\frac{2N-1}{2}\right)} \quad if\;\; N=1\; \_\; \exp_2 = \frac{-2\lambda}{MR_e}$$

$$if\;\; N=N\; \_\; \exp_2 = \frac{-\lambda}{R_e\left(1-\frac{1}{2N}\right)}$$

Further, since $r = R_e\left[1 - \frac{(2N-1)}{2M}\right]$ 162

$$M_{(m)} = 4\pi\rho\left(\frac{R_e^3}{M}\right)\left[1-\frac{(2N-1)}{2M}\right]^2$$

If we choose M such that $\frac{R_e}{M} >> \lambda$ 164 then we may write

$$F_{(m)} = \left(\frac{-Gm}{R_e^2}\right) 4\pi\rho \left(\frac{R_e^3}{M}\right)\left[1 - \frac{(2m-1)}{2M}\right]^2 \left\{ \frac{1}{2}\left[2 - \frac{\lambda}{R_e\left[2 - \frac{(2m-1)}{M}\right]} - \frac{\lambda}{R_e\left(\frac{2m-1}{M}\right)}\right]\right.$$

$$+ \frac{\lambda}{R_e\left[2 - \frac{(2m-1)}{M}\right]}\left\{ \log\left[\frac{2m-1}{4M-(2m-1)}\right]\right.$$

$$\left.\left. + \sum_{N=1}^{\infty} \frac{(-\lambda)^N (2M)^N}{N.N!\,R_e^N}\left[\frac{(2m-1)^N - 1}{(2m-1)^N [4M - 2m - 1]^N}\right]\right\}\right\}$$

Suppose we look at three shells such that M = 3 then we find

$$F_1 = \frac{-GM}{R_e^2}\left\{4\pi\rho\left(\frac{R_e^2}{3}\right)\left[1 - \frac{1}{6}\right]\right\}\left\{1 - \frac{\lambda}{2R_e\left[2 - \frac{1}{3}\right]} - \frac{\lambda}{2R_e\left(\frac{1}{3}\right)}\right.$$

$$\left. + \frac{\lambda}{R_e\left[\frac{5}{3}\right]}\left\{\log\left[\frac{1}{12-1}\right] + \sum_{N=1}^{\infty}\frac{(-\lambda)^N 6^N}{N.N!\,R_e^N}\left[\frac{1^0 - 1}{[11]^N}\right]\right\}\right\}$$

$$= \left[\frac{-GM_1 m}{R_e^2}\right]\left\{1 - \frac{3\lambda}{2 \bullet 5 R_e} - \frac{3\lambda}{2 \bullet R_e} - \frac{2.398)3\lambda}{5}\right\}$$

$$F_1 = \left[\frac{-GM_1 m}{R_e^2}\right]\left\{1 - \frac{4.198\lambda}{R_e}\right\} \; if \;\; M_1 = 4\pi\rho\left(\frac{R_e^3}{3}\right)\left(\frac{5}{6}\right)^2$$

or

$$F_{out} = \left(\frac{-GMm}{R_e^2}\right)\left\{1 - 3.548\frac{\lambda}{R_e}\right\}$$

Now we can do the same sort of thing for the "inside the shell" force. This case has the lower limits on x changed so that

$$F_{in} = \frac{-\pi G\rho rtm}{R^2}\int_{x=r-R}^{R+r}\frac{d}{dx}\left[\frac{f(x)e^{-\frac{\lambda}{x}}}{x}\right]dx \quad , where\; x = r - R \geq \frac{t}{2}$$

with

$$\frac{f(x)}{x}e^{-\frac{\lambda}{x}}=\left[\frac{-(R^2 1 r^2}{x}+x\right]e^{-\frac{\lambda}{x}}+2\lambda\left\{\log\left(\frac{1}{x}\right)+\sum_{N=1}^{\infty}\frac{(-\lambda)^N}{N.N!\,x^N}\right\}$$

Then for a shell with R_e as the inner radius and a thickness of t then $r=R_e=t/2$ and we desire to know what the gravitational effort for $R=R_e$. Then we have $R+r=2R_e+t/2$ and $r-R=t/2$ then

$$F_{in}=\left[\frac{-\pi G\rho\left(R_e+\frac{t}{2}\right)tm}{R_e^2}\right]\left\{\frac{-\left[R_e^2-\left(R_e+\frac{t}{2}\right)^2\right]}{\left(2R_e+\frac{t}{2}\right)}e^{-\frac{\lambda}{2R_e+\frac{t}{2}}}\right.$$

$$-\frac{\left[R_e^2-\left(R_e+\frac{t}{2}\right)^2\right]}{\frac{t}{2}}e^{\frac{\lambda}{\frac{t}{2}}}+\left[2R_e+\frac{t}{2}\right]e^{-\frac{\lambda}{2R_e+\frac{t}{2}}}-\frac{t}{2}e^{-\frac{\lambda}{\frac{t}{2}}}$$

$$\left.+2\lambda\left\{\frac{\frac{t}{2}}{2R_e+\frac{t}{2}}+\frac{(-\lambda)^N}{N.N!}\left[\frac{1}{\left[2R_e+\frac{t}{2}\right]}-\frac{1}{\left(\frac{t}{2}\right)^N}\right]\right\}\right\}$$

if $\lambda>>2R_e$, $R_e>>t/2$, then we have the approximation

$$F_{in}\cong\left(\frac{-GMm}{R_e^2}\right)\left\{\left(\frac{t}{4}\right)\left[\frac{\left(1-\frac{\lambda}{2R_e}\right)}{2R_e}+\frac{2e^{\frac{-2\lambda}{t}}}{t}\right]\right.$$

$$+\left(\frac{\lambda}{2R_e}\right)\log\left(\frac{t}{4R_e}\right)+\frac{\lambda}{2R_e}\sum_{N=1}^{\infty}\frac{(-\lambda)^N}{N.N!}\left[\frac{1}{(2R_e)^N}-\frac{2^N}{t^N}\right]$$

Example: Compare the change in gravitational field as one goes from the Earth's surface down a deep well to a depth of d. The Newtonian gravitational strength at the earth's surface is

$$F_N=\frac{-GM_e m}{R_e^2}$$

For the neo-Newtonian force we must see the same force, thus, we must set

$$F_{NN}=\frac{-GM_e m}{R_e^2}\cong\frac{-GMm}{R_e^2}\left(1-3.55\frac{\lambda}{R_e}\right)$$

or

$$M = \frac{M_e}{\left(1 - 3.55\frac{\lambda}{R_e}\right)}$$

Now the Newtonian gravitation at a depth of d.

$$F_N(d) = \frac{-GM_e m}{(R_e - d)^2} = \frac{-GM_e m}{(R_e^2 - 2d\,R_e - d^2)}$$

or

$$F_N(-d) \cong \frac{-GM_e m}{R_e^2\left(1 - \frac{2d}{R_e}\right)}$$

The variation in force from the surface to the depth is then

$$\Delta F_N \cong \frac{-GM_e m}{R_e^2} + \frac{GM_e m}{R_e^2\left(1 - \frac{2d}{R_e}\right)} = \frac{GM_e m}{R_e^2}\left[\frac{1 - 1 + \frac{2d}{R_e}}{\left(1 - \frac{2d}{R_e}\right)}\right]$$
$$= \left(\frac{GM_e m}{R_e^2}\right)\left[\frac{GM_e m}{R_e^2}\right]$$

or

$$\Delta F_N \cong \left(\frac{2d}{R_e}\right)\left[\frac{GM_e m}{R_e^2}\right].$$

On the other hand, the gravitation force in the well with the neo-Newtonian potential would be given by

$$F_{NN}(-d) = \frac{-GM_e m}{R_e^2}\left\{1 + \frac{M}{M_e}\frac{\lambda}{4R_e} - \frac{M}{2M_e}\left[1 - e^{-\frac{2\lambda}{t}}\right] - \frac{\lambda M}{2M_e R_e}\log\left(\frac{t}{4R_e}\right)\right.$$
$$\left. - \frac{\lambda M}{2M_e R_e}\sum_{N=1}^{\infty}\frac{(-2\lambda)^N}{N.N!}\left[\frac{1}{(4R_e)^N} - \frac{1}{t^N}\right]\right\}$$

then $\Delta F_{NN} = F_{NN}(R_e) - F_{NN}(R_e\text{-}d)$

$$\Delta F_{NN} = \frac{-GM_e m}{R_e^2} + \frac{GM_e m}{(R_e - 4)^2}\left\{1 + \frac{M}{M_e}\left\{\frac{\lambda}{4(R_e - d)} - \frac{1}{2}\left[1 - e^{-\frac{2\lambda}{d}}\right] - \frac{\lambda}{2R_e}\log\left(\frac{d}{4R_e}\right)\right.\right.$$
$$\left.\left. - \frac{\lambda}{2(R_e - d)}\sum_{N=1}^{\infty}\frac{(-2\lambda)^N}{N.N!}\left[\frac{1}{(4R_e)^N} - \frac{1}{d^N}\right]\right\}\right\}$$

but $\frac{M}{M_e} = \frac{1}{\left(1 - 3.55\frac{\lambda}{R_e}\right)}$ 181 so that for d = 5,500 ft = 1,676m and R_e = 6.4 x 10^6m then

$$\Delta F_{NN} \cong \left(\frac{2d}{R_e}\right)\left[\frac{GM_e m}{R_e^2}\right]\left\{1 + \left(\frac{M}{M_e}\right)\left\{-954\left[1 - e^{-\frac{2\lambda}{d}}\right] + 1.512 \times 10^{-3}\lambda\right.\right.$$
$$\left.\left. - 1.492 \times 10^{-4}\lambda\sum_{N=1}^{\infty}\frac{(-2\lambda)^N}{N.N!}\left[\frac{1}{(2.56 \times 10^7)^N} - \frac{1}{(1676)^N}\right]\right\}\right\}$$

Now

$$\sum_{N=1}^{\infty}\frac{(-2\lambda)^N}{N.N!}\left[\frac{1}{(2.56 \times 10^7)^N} - \frac{1}{(1676)^N}\right] = \begin{array}{l} \frac{+2\lambda}{1} 5.966 \times 10^{-4} \rightarrow 1.193 \times 10^{-3}\lambda \\ \frac{+4\lambda^2}{2.2}(-3.56 \times 10^{-7} \rightarrow -3.56 \times 10^{-7}\lambda^2 \\ \frac{-8\lambda^3}{3 \bullet 7 \bullet 3}(-2.124 \times 10^{-10} \rightarrow +9.44 \times 10^{-11}\lambda^3 \\ \frac{+16\lambda^4}{4 \bullet 2 \bullet 3 \bullet 4}(-1.267 \times 10^{-13} \rightarrow 2.11 \times 10^{-14}\lambda^4 \end{array}$$

Thus, we have

$$\Delta F_{NN} \cong \left(\frac{2d}{R_e}\right)\left(\frac{GM_e m}{R_e^2}\right)\left(1 - \frac{3.548\lambda}{1676}\right)$$

$$\Delta F_{NN} \cong \left(\frac{2d}{R_e}\right)\left(\frac{GM_e m}{R_e^2}\right)\left(1 - \frac{3.548\lambda}{d}\right)$$

$$F_N(-d) = \frac{-GM_e m}{R_e^2\left(1 - \frac{2d}{R_e}\right)}$$

But the neo-Newtonian gravitational force at the bottom of a well of depth d is given by

$$F_{NN}(-d) = \frac{-GMm}{R_e^2\left(1-\frac{2d}{R_e}\right)}\left[1-3.548\frac{\lambda}{R_e}\right]$$

$$+\frac{-GM_e m}{R_e^2\left(1-\frac{2d}{R_e}\right)}\left[1-3.548\frac{\lambda}{R_e}\right]^2$$

$$F_{NN}(-d) = \frac{-GM_e m}{R_e^2\left(1-\frac{2d}{R_e}\right)}\left[1-7.096\frac{\lambda}{R_e}\right]$$

which may be written

$$F_{NN}(-d) = F_N(-d)\left(1-7.096\frac{\lambda}{R_e}\right)$$

This gives the first order approximation of the deviation from Newtonian gravitation predicted by the neo-Newtonian potential and shows that the predicted gravitational force of the Earth decreases more rapidly than Newtonian gravitation does.

5.5 Inertial and Gravitational Mass and their Equivalence

There are three ways in which mass appears in Newton's Second Law when gravitational forces are considered. Consider his gravitational force law which may be written

$$\overline{F} = -\frac{Gm_1 m_2}{r^3}\overline{r}$$

and his Second Law which is

$$\overline{F} = \frac{d}{dt}(m\overline{v}) = m\frac{d^2\overline{r}}{dt^2}\ .$$

In these equations there are the inertial mass, m, and two gravitational masses, m_1 and m_2. The force equation comes from considering the force on m_2 due to the the gravitational field of m_1. In this case m_1 is usually referred to as the active gravitational mass while m_2 is the passive gravitational mass. Classically Newton's Third Law is imposed in order to show that the ratios of active and passive gravitational masses must be equal. Consider

$$\overline{F}_1 = -\overline{F}_2$$

so that

$$Gm_{1a}m_{2p} = Gm_{2a}m_{1p}$$

where the subscripts a and p refer to the mass's role as either an active or passive gravitational mass. This leads us to the equation

$$\frac{m_{1a}}{m_{2a}} = \frac{m_{1p}}{m_{2p}}$$

which means that since the ratios must be equal the m_a and m_p may be made equal.

The equality of inertial and gravitational mass is not predictable by Newton's laws. Rather, it is taken as an assumption. This assumption has been subjected to increasingly accurate experimentation by Eotvos in the 1880's, by Dicke in 1964, and by Braginski in 1971. The present limit of comparision between gravitational and inertial mass in about one part in 10^{12}.

Now let's consider these same three mass concepts in the context of the Dynamic Theory. First, there is the inertial mass density. It makes its appearance in Section 3.1 when we impose the principle of increasing entropy as a variational principle. The metric element is given in terms of the specific entropy while the entropy principle is in terms of the entropy density. The effect of this is to introduce the mass density as a product of the acceleration into the equations of the force densities (see Eqn. (3.5)). The same inertial mass concept leads to the Einstein energy and mass relation in Section 3.2.

The other two mass concepts enter first through the field equations given by Eqn.s (3.15) and from them the force densities in Eqn.s (3.17). In Section 5.1 we went through the field equations to determine the charge-to-mass conversion needed to keep the units consistant. Here we found that the passive gravitational mass given by Eqn. (5.12) was

$$M \cong m_p\sqrt{\varepsilon G} \tag{5.76}$$

while the gravitational field associated with a gravitational mass is given by Eqn. (5.10), when the evaluated parameters are used, as

$$V_r = m_a \sqrt{\frac{G}{\varepsilon}} \left[1 + \dot{G}t\right] \left[\frac{\left(1 - \frac{\lambda_a}{r}\right) e^{-\frac{\lambda_a}{r}}}{r^2} \right]$$

(5.77)

where the mass in the gravitational field equation is to be considered the acitve mass and therefore we've used the subscript a to denote this. The gravitational force due to the passive mass M being in the gravitational field V_r is then

$$\overline{F}_{12} = -Gm_{1p} m_{2a} (1 + \dot{G}t) \left[\frac{\left(1 - \frac{\lambda_{2a}}{r}\right) e^{-\frac{\lambda_{2a}}{r}}}{r^3} \right] \overline{r}_{12} . \tag{5.78}$$

By looking at Eqn. (5.78) we may see that, first, it is the active gravitational mass that has the time dependence and not the passive gravitational mass. Further, only when the active gravitational masses are identical with their λ's the same will the active and passive gravitational masses be equal. We've used the subscripts 12 in the force equation to denote the force on mass 1 in the pressence of the field of mass 2. We may consider the force on mass 1 when in the field of mass 2 and we find

$$\overline{F}_{21} = Gm_{2p} m_{1a} (1 + \dot{G}t) \left[\frac{\left(1 - \frac{\lambda_{1a}}{r}\right) e^{-\frac{\lambda_{1a}}{r}}}{r^3} \right] \overline{r}_{12} . \tag{5.79}$$

If we form the ratio of Eqn.s (5.78) and (5.79) we find that

$$\frac{\overline{F}_{12}}{-\overline{F}_{21}} = \frac{m_{1p} m_{2a} \left[\left(1 - \frac{\lambda_{1a}}{r}\right) e^{-\frac{\lambda_{1a}}{r}} \right]}{m_{2p} m_{1a} \left[\left(1 - \frac{\lambda_{2a}}{r}\right) e^{-\frac{\lambda_{2a}}{r}} \right]} . \tag{5.80}$$

Further, only for identical gravitational masses will Newton's Third Law be satisfied within the Dynamic Theory.

5.6 Cosmology

The hot big bang model of the Universe is the model which is in vogue now. Virtually all the journals print numerous articles relating to some aspect of the hot big bang model. The model is based upon the

Newtonian Gravitational and the notion of a scale of the universe that is changing with time. This notion is borrowed from Einstein's General Theory of Relativity, however Einstein's theory is not used in the hot big bang model itself. It would seem a shame to discuss a new gravitational potential such as presented in this book without some discussion of its possible impact upon the hot big bang model. It would have been preferable to wait until the entire solution could be presented. However, this is not possible now so this presentation will include a discussion of how one might expect the new potential to impact the hot big bang model and the problems that render the solution illusive.

The development of the standard big bang model begins with considering a spherical piece of the universe with an observer at the center. This sphere is considered to be filled with "dust" of density $\rho(t)$ with a galaxy of interest placed at the outer boundary of the sphere which has a radius denoted by x. When Gauss's law and Newton's laws of motion and gravitation is used one arrives at

$$m_g \frac{d^2 x}{dt^2} = -\frac{4\pi}{3} \frac{x^3 \rho(t) G m_g}{x^2} = -\frac{4\pi}{3} G \rho(t) m_g x. \tag{5.81}$$

But consider what happens if one wishes to compare this with the non-singular potential of the Dynamic Theory. Then Eqn. (5.81) becomes

$$m_g \frac{d^2 x}{dt^2} = -\frac{4\pi}{3} \frac{x^3 \rho(t) G m_g}{x^2} \left(1 - \frac{\lambda}{x}\right) e^{-\frac{\lambda}{x}} = -\frac{4\pi}{3} G \rho(t) x \left(1 - \frac{\lambda}{x}\right) e^{\frac{\lambda}{x}}.$$

Now let us replace x with the comoving coordinate x=R(t)r where R(t) is the scale factor of the universe and r is the comoving distance coordinate as is done in the standard model. When we also normalize the density to its value at the present epoch, ρ_o , by $\rho(t)=\rho_o R^{-3}(t)$ we obtain

$$\frac{d^2 R}{dt^2} = -\frac{4\pi G \rho_o}{3} \left(1 - \frac{\lambda}{Rr}\right) \frac{e^{-\frac{\lambda}{Rr}}}{R^2}. \tag{5.83}$$

We can begin to see the trend to be expected from the universe from Eqn. (5.83) by noting that should we look back in time to the point when $R=r/\lambda$ then we would have a point in time, say T_1 when the acceleration of the universe would have been zero. At times before T_1 there would have been an acceleration outward while for times after T_1,such as the current epoch, the rate of the expansion of the universe is slowing down. This is a very different story than is told from by the standard model. But how is it different? It is the same as the standard model in that from Eqn. (5.83) one sees that the universe was forced into expansion at early times and is now slowing down its rate of expansion. One big difference between the story to be told by Eqn. (5.83) and the standard model is that Eqn. (5.83) gives the reason for the initial expansion and it denies that the universe was ever collapsed to a singular point as supposed by the standard model. To better see the first contention we should proceed a little further.

If we multiply Eqn. (5.83) by dR/dt and integrate with respect to

$$\dot{R}^2 = \frac{8\pi G}{3}\left[\rho(t) R^2 e^{-\frac{\lambda}{Rr}} + \frac{\varepsilon(t) R^2}{2c^2}\right] - kc^2. \qquad (5.84)$$

time we find

In Eqn. (5.84) we have included the term for the radiation for completeness. Now let us evaluate the constant of integration, k, by setting the values of R, dR/dt, ρ, and ε at their present day values of 1, H_o, ρ_o, and ε_o. Then Eqn. (5.84) becomes

$$H_o^2 = \frac{8\pi \rho_o G}{3} e^{-\frac{\lambda}{r}} + \frac{4\pi G \varepsilon_o}{3c^2} - kc^2. \qquad (5.85)$$

If we now make the definitions

$$\rho_c \equiv \frac{3H_o^2}{8\pi G} \;\; and \;\; \Omega \equiv \frac{\rho_o e^{-\frac{\lambda}{r}}}{\rho_c},$$

Eqn. (5.85) may be put into Eqn. (5.84) to obtain

$$\dot{R}^2 = H_o^2\left(\frac{\Omega e^{-\frac{\lambda}{Rr}}}{R e^{-\frac{\lambda}{r}}} + 1 - \Omega\right) + \frac{H_o^2 \varepsilon_o}{2c^2 \rho_c}\left(\frac{1}{R^2} - 1\right). \qquad (5.86)$$

We may now take a look at some of the implied dynamics from Eqn. (5.86).

First look at the dynamics as R tends to infinity and there is no radiation. For this case we would have

$$\dot{R}_\infty^2 = H_o^2(1 - \Omega),$$

which is the same as in the standard model.

Now suppose we look backward in time to the time when dR/dt was zero? Then Eqn. (5.86) becomes

$$0 = \rho_o R e^{-\frac{\lambda}{Rr}} - \rho_o R^2 e^{-\frac{\lambda}{r}} + \rho_c R^2 + \frac{\varepsilon_o}{2c^2}\left(1 - R^2\right). \qquad (5.87)$$

This is a transcendental equation which could be solved for R if we knew λ, r and the density of the dust and radiation at the current epoch. It may be seen from Eqn. (5.87) that if there is no radiation and R does not equal zero then

$$R = \frac{e^{-\frac{\lambda}{Rr}}}{\left[e^{-\frac{\lambda}{r}} - \frac{\rho_c}{\rho_o}\right]}. \tag{5.88}$$

There is also a trivial solution at R=0 in Eqn. (5.87) but for this case the acceleration is also zero and therefore no dynamics are allowed.

While at first glance it may appear that we have as good a developed solution as is arrived at in the standard model, consider the following points. Our galaxy was considered to be on the outside limit of a sphere of dust. For the current epoch the density of the dust is very small locally to the galaxy and Gauss's law for considering the total mass of the sphere of dust to be placed at the center should hold very well. But what about when we are looking back in time when the density was a lot greater. At some density we are no longer able to approximate result used in Eqn. (5.82) but must use the solution developed in the discussion of the Fifth Force. Then our conclusions arrived at above are only good in a general sense and are not quantitatively accurate.

A second point concerns the fact that we have developed the gauge function in prior sections. If this is the scale of the universe as the gauge function is supposed to be, then why are we again trying to solve for it here? If the scale of the universe is given by the gauge function then the dynamics may be over specified if we put the radiation into the equation for the acceleration such as Eqn. (5.83). On the other hand, what is the source for the radiation? If there is no hot big bang for the radiation to come from where might it originate? The Dynamic Theory displays an inductive coupling between the electromagnetic and the gravitational fields. could the radiation be due to the expansion of the gravitating mass of the universe? If so then a knowledge of the gauge function might turn the equation of motion for our galaxy into a prediction of the radiation required at the present time. This prediction might then be compared to the measured radiation. However, there is the necessity to have a λ for the universe. It may be obtained from the gauge function also as GM/c^2. But how is M determined?

Perhaps this is sufficient to point out that the overall picture of cosmology to be given by the Dynamic Theory is not yet complete but in any event will likely be very different from the hot big bang model of the universe. Will it allow for high temperatures needed for accounting for the abundances of the elements? Since it allow for the universe to be much smaller in the past it would have the associated high temperatures. Yet it should not have the infinite temperatures associated with a singular universe.