Chapter 6 Electromagnetogravitic Waves

Given the system of equations, Eqn. (3.15), and the interpretations that; E is the electric field, B is the magnetic field, V is the gravitational field, and V_4 is the gravitational potential, then the question arises as to how the electromagnetogravitic waves may propagate.

6.1 Wave Equations

The usual assumptions such as $\partial\mu/\partial\gamma = \partial\varepsilon/\partial\gamma = 0$ and $\rho = 0$ may be used to derive the wave equations for the four field quantities. Other assumptions used are that the media is isotropic and that $J = \sigma E$ and $J_4 = \sigma_4 V_4$. The resulting wave equations are

$$\Delta^2 \overline{V} - \left(\frac{4\pi\mu\sigma_4}{c^2}\right)\frac{\partial \overline{V}}{\partial t} - \left(\frac{\mu\varepsilon}{c^2}\right)\frac{\partial^2 \overline{V}}{\partial t^2} + a_0^2\frac{\partial^2 \overline{V}}{\partial \gamma^2} = \frac{a_0 4\pi\mu}{c}\left[\frac{\partial(\sigma \overline{E})}{\partial\gamma} - \sigma_4\frac{\partial \overline{E}}{\partial\gamma}\right], \quad (6.1)$$

$$\Delta^2 \overline{E} - \left(\frac{4\pi\mu\sigma}{c^2}\right)\frac{\partial \overline{E}}{\partial t} - \left(\frac{\mu\sigma}{c^2}\right)\frac{\partial^2 \overline{E}}{\partial t^2} + a_0^2\frac{\partial^2 \overline{E}}{\partial \gamma^2} = 0, \quad (6.2)$$

$$\Delta^2 \overline{B} - \left(\frac{4\pi\mu\sigma}{c^2}\right)\frac{\partial \overline{B}}{\partial t} - \left(\frac{\mu\varepsilon}{c^2}\right)\frac{\partial^2 \overline{B}}{\partial t^2} + a_0^2\frac{\partial^2 \overline{B}}{\partial \gamma^2} = 0, \quad (6.3)$$

and

$$\Delta^2 V_4 - \left(\frac{4\pi\mu\sigma_4}{c^2}\right)\frac{\partial V_4}{\partial t} - \left(\frac{\mu\varepsilon}{c^2}\right)\frac{\partial^2 V_4}{\partial t^2} + a_0^2\frac{\partial^2 V_4}{\partial \gamma^2} = 0. \quad (6.4)$$

The inhomogeneous term in Eqn. (6.3) displays an interconnection between the electric and gravitational waves. Further, this term produces the question of whether the propagation vector for the gravitational wave can be the same as the propagation vector for the electric wave. In Maxwellian electromagnetism it may be shown that the propagation vector for the electric wave must be the same as the propagation vector for the magnetic wave. However, this is not true, in general, for this system of equations.

6.2 Wave Solutions

Given that the propagation vectors may be different a trial solution may be sought during which the conditions for identical propagation vectors may be exposed.

If the waves are considered to be propagating in the positive x-direction, then the trial solutions may be taken to be of the form

$$\overline{E} = \overline{E_0} \exp\left[-i\left(\omega t - k_e x - k_{4e}\gamma\right)\right] \ ,$$
$$\overline{B} = \overline{B_O} \exp\left[-i\left(\omega t - k_b x - k_{4b}\gamma\right)\right] \ ,$$
$$\overline{V} = \overline{V_O} \exp\left[-i\left(\omega t - k_v x - k_{4v}\gamma\right)\right] \ ,$$

and

$$V_4 = V_{40} \exp\left[-i\left(\omega t - k_4 x - k_{44}\gamma\right)\right] \ , \quad (6.5)$$

By making the definitions

$$\begin{aligned} A_e &= a_0 c k_{4e} \ , \\ A_b &= a_0 c k_{4b} \ , \\ A_v &= a_0 c k_{4v} \ , \\ A_4 &= a_0 c k_{44} \ , \end{aligned} \quad (6.6)$$

and substituting the trial solutions, Eqn. (6.5) into the wave equations, Eqn.s (6.1)- (6.4), we obtain the indicial relations:

$$\left(k_{v0} c\right)^2 = \mu\varepsilon\,\omega^2 + i4\pi\mu\omega\,\sigma_4 - A_v^2 + i4\pi\mu A e\left[\left(\sigma_4 - \sigma\right) + \left(\frac{i}{k_{4e}}\right)\frac{\partial\sigma}{\partial\gamma}\right]\left(\frac{E_\alpha}{V_\alpha}\right)$$

$$\begin{aligned} \left(k_e c\right)^2 &= (kc)^2 - A_e^2 \ , \\ \left(k_b c\right)^2 &= (kc)^2 - A_b^2 \ , \end{aligned} \quad (6.7)$$

and

$$\left(k_4 c\right)^2 = \mu\varepsilon\,\omega^2 + i4\pi\mu\omega\,\sigma_4 - A_4^2 \ ,$$

where α = 1, 2, 3, and $(kc)^2 = \mu\varepsilon\omega^2 + i4\pi\mu\omega\sigma$.

Substituting the trial solutions into the continuity equation of Eqns. (3.14), we find

$$\left(\sigma - \sigma_4\right) = \left(\frac{-i}{k_{44}}\right)\frac{\partial\sigma_4}{\partial\gamma} \ . \quad (6.8)$$

The ratio (E_α/V_α) appearing in Eqn. (6.7) indicates that we need to know the relationship between the components of the V field and the components of the E field. These relationships may be found by substituting the trial solution into each of Eqn. (3.15). In this substitution, the further limiting assumption that the electric field may be polarized so the $E_z = 0$ and $E_x = 0$ is made in order to simplify the solution. It should be pointed out though that, in contradistinction with Maxwellian electrodynamics, E_x is not required to be zero by the differential equations.

The differential equations require the following relationship among the non-zero components, given the trial solutions and the imposed restrictive assumptions:

$$B_z = \left(\frac{k_e c}{\omega}\right) E_y \ , \quad (6.9)$$

and

$$V_y = \left(\frac{-A_e}{\omega}\right) E_y = \left(\frac{A_b}{\omega}\right)\left(\frac{k_e}{k_v}\right) E_y \ .$$

Thus, the imposed assumptions reduce the solution to only three non-zero components, E_y, B_z, and V_y.

If we consider the different expressions, from Eqn. (6.9), for B_z, and take the partial derivative with respect to the mass density we find

$$\frac{\partial B_z}{\partial \gamma} = \frac{\partial \left(\frac{k_e c}{\omega} \right) E_y}{\partial \gamma}$$

requires that

$$\left(\frac{1}{k_e} \right) \frac{\partial k_e}{\partial \gamma} = \left(\frac{i}{a_0 c} \right) \left(A_b - A_e \right) \quad . \qquad (6.10)$$

From Eqn. (6.9) we also find

$$A_b k_e = A_e k_v \quad , \qquad (6.11)$$

while differentiating with respect to γ-produces

$$\left(\frac{1}{A_e} \right) \frac{\partial A_e}{\partial \gamma} = \left(\frac{i}{a_0 c} \right) \left(A_v - A_e \right) \quad . \qquad (6.12)$$

With the assumption that there is no longitudinal field component the surviving system of equations is

$$\begin{aligned} (k_e c)^2 &= (kc)^2 - A_e^2 \quad , \\ (k_b c)^2 &= (kc)^2 - A_b^2 \quad , \qquad (6.13) \\ (k_v c)^2 &= (kc)^2 - A_v^2 + \left(\frac{4\pi\mu\omega a_0 c}{A_e} \right) \frac{\partial \sigma}{\partial \gamma} \quad , \\ k_v &= \left(\frac{A_b}{A_e} \right) k_e \quad , \\ \left(\frac{1}{k_e} \right) \frac{\partial k_e}{\partial \gamma} &= \left(\frac{i}{a_0 c} \right) \left(A_b - A_e \right) \qquad (6.14) \\ \left(\frac{1}{A_e} \right) \frac{\partial A_e}{\partial \gamma} &= \left(\frac{i}{a_0 c} \right) \left(A_v - A_e \right) \end{aligned}$$

where the definition

$$(kc)^2 = \mu\varepsilon\,\omega^2 + i4\pi\mu\omega\sigma \qquad (6.15)$$

has been used.

The partial derivative, $\partial\sigma/\partial\gamma$, appearing in Eqn. (6.14) may be found from experiment in the following manner; because

$$\frac{\partial \sigma}{\partial \gamma} = \left(\frac{\partial \sigma}{\partial T} \right) \left(\frac{\partial T}{\partial \gamma} \right)$$

where T is the temperature. The conductivity, σ, is the reciprocal of the resistivity, θ, whose linear dependence upon the temperature is given by

$$r = r_0 \left[1 + \bar{\alpha} \left(T - T_0 \right) \right]$$

Then

$$\frac{\partial \sigma}{\partial T} = \frac{\partial \left(\frac{1}{r} \right)}{\partial T} = -\sigma^2 r_0 \bar{\alpha} \quad .$$

On the other hand, the coefficient of volume expansion is defined as

$$\beta = \frac{1}{(vol.)} \frac{\partial (vol.)}{\partial T} \quad ,$$

but

$$\gamma = \frac{mass}{vol.} \quad ,$$

Therefore,

$$\frac{\partial \gamma}{\partial T} = \gamma\beta \quad .$$

Thus,

$$\frac{\partial \sigma}{\partial \gamma} = \frac{r_0 \bar{\alpha} \sigma^2}{\beta\gamma} \quad . \quad (6.15)$$

The solution of Eqn. (6.15) is

$$\sigma = \frac{\gamma\left(\frac{\beta}{r_0 \bar{\alpha}}\right)}{l_n\left(\frac{\gamma_0}{\gamma}\right)} \qquad (6.16)$$

where γ_0 is the mass density evaluated at the temperature T_0.

Given the expression for $\partial\sigma/\partial\gamma$, it may be seen that the system of Eqns. (6.13) and (6.14) represent six-equations in the six unknowns, A_e, A_b, A_v, k_e, k_b, and k_v. The system may be solved as shown in the following.

The unknown, k_b, appears only in one equation; therefore, this equation may be considered to determine k_b once the solution for the other unknowns are determined.

Eqn. (6.11) may be used to eliminate A_b, in Eqn. (6.10) leaving us with four equations in four unknowns. Eqn. (6.10) now looks like

$$k_v = k_e - \left(\frac{ia_0 c}{A_e}\right)\frac{\partial k_e}{\partial \gamma} \qquad (6.17)$$

Eqn. (6.13) may be used to determine A_v by rewriting it as

$$A_v = A_e - \left(\frac{ia_0 c}{A_e}\right)\frac{\partial A_e}{\partial \gamma} \quad . \quad (6.18)$$

This may be used to eliminate A_v from the third of Eqn.s (6.7) leaving three equations in three unknowns.

By differentiating the first of Eqn. (6.7) with respect to the mass density, it becomes

$$2 k_e c^2 \frac{\partial k_e}{\partial \gamma} = i4\pi\mu\omega \frac{\partial \sigma}{\partial \gamma} - 2 A_e \frac{\partial A_e}{\partial \gamma} \quad . \qquad (6.19)$$

Substituting Eqns. (6.17) and (6.18) into the third Eqn. (6.7) and using Eqn. (6.19) results, after some manipulation, in

$$(kc)^2\left[\left(\frac{1}{A_e}\right)\frac{\partial A_e}{\partial \gamma}\right]^2 - 2\left(i2\pi\mu\omega\frac{\partial \sigma}{\partial \gamma}\right)\left[\left(\frac{1}{A_e}\right)\frac{\partial A_e}{\partial \gamma}\right] + \left(i2\pi\mu\omega\frac{\partial \sigma}{\partial \gamma}\right)^2\left(\frac{1}{A_e}\right)^2 = 0. \qquad (6.20)$$

This is a quadratic equation in $\partial A_e/\partial\gamma$ whose solutions are

$$\frac{\partial A_e}{\partial \gamma} = \frac{i2\pi\mu\omega\frac{\partial \sigma}{\partial \gamma}}{(kc)^2}\left[A_e \pm \sqrt{A_e^2 - (kc)^2}\right]$$

or

$$\frac{\partial A_e}{\partial \sigma} = \left[\frac{i2\pi\mu\omega}{(kc)^2}\right]\left[A_e \pm \sqrt{A_e^2 - (kc)^2}\right] .$$

Therefore we have

$$\frac{dA_e}{\left[A_e \pm \sqrt{A_e^2 - (kc)^2}\right]} = \frac{d(kc)}{(kc)} .$$

(6.21)

But from the definition of $(kc)^2$ we find that

$$d(kc)^2 = i4\pi\mu\omega d\sigma$$

Thus,

$$\frac{i2\pi\mu\omega d\tau}{(kc)^2} = \frac{d(kc)^2}{2(ki)^2} = \frac{2(kc)d(kc)}{2(kc)^2} = \frac{d(kc)}{ki} .$$

Eqn. (6.21) now becomes

$$\frac{d\,A_e}{\left[A_e \pm \sqrt{A_e^2 - (kc)^2}\right]} = \frac{d(kc)}{(kc)} .$$

(6.22)

By using the method of substitution, recognizing that it may be put into a homogeneous form, and realizing the solution may be complex, we arrive at the solution of Eqn. (6.22) as

$$A_e = \left(\frac{1}{2c_2}\right)\left[(kc)^2 c_2^2 - 1\right]$$

(6.23)

where c_2 is a constant of integration such that

$$\frac{\partial c_2}{\partial \sigma} = 0 ,$$

that is, c_2 may depend upon μ, ε, and ω but not σ.

Eqn. (6.23) is a quadratic equation in c_2 and may be solved yielding

$$c_2 = \frac{A_e +_- \sqrt{A_E^2 + (kc)^2}}{(kc)^2} \quad .$$

(6.24)

Because c_2 does not depend upon σ it is unaffected by setting $\sigma = 0$, so if $A_{eo} = A_e$ ($\sigma = 0$) then

$$c_2 = \frac{A_{eo} +_- \sqrt{A_{eo}^2 + \mu\varepsilon\,\omega^2}}{\left(\mu\varepsilon\,\omega^2\right)} \quad .$$

(6.25)

By substituting Eqn. (6.25) back into Eqn. (6.23) we find the sign before the radical must be taken to be positive. Then the expression for A_e becomes

$$A_e = A_{eo} + i4\pi\mu\omega\sigma h \quad .$$

(6.26)

where

$$h \equiv \left\{ \frac{A_{eo}}{\mu\varepsilon\,\omega^2} + \frac{1}{2\left[A_{eo} + \sqrt{A_{eo}^2 + \mu\varepsilon\,\omega^2}\right]} \right\} \quad .$$

Using Eqn. (6.26) in the system of equations, Eqn. (6.14), we may, after a great deal of algebra, write the total solution as:

$$A_e = A_{eo} + i4\pi\mu\omega\sigma h \quad ,$$

(6.27)

where

$$h \equiv \left\{ \frac{A_{eo}}{\mu\varepsilon\,\omega^2} + \frac{1}{2\left[A_{eo} + \sqrt{A_{eo} + \mu\varepsilon\,\omega^2}\right]} \right\} \quad .$$

Now we may write

$$k_e = \alpha_e + i\,\beta_e \, ,$$

with

$$\alpha_e = \sqrt{\frac{\mu\tilde{\varepsilon}}{2}}\left\{\sqrt{1+\left(\frac{4\pi\tilde{\sigma}}{\tilde{\varepsilon}\omega}\right)^2}+1\right\}^{\frac{1}{2}}$$

$$\beta_e = \left(\frac{\omega}{c}\right)\sqrt{\frac{\mu\tilde{\varepsilon}}{2}}\left\{\sqrt{1+\left(\frac{4\pi\tilde{\sigma}}{\tilde{\varepsilon}\omega}\right)^2}-1\right\}^{\frac{1}{2}} , \tag{6.28}$$

$$\tilde{\varepsilon} = \varepsilon\left[1-\frac{A_{eo}}{\mu\varepsilon\omega^2}+\frac{(4\pi\mu\omega\sigma h)^2}{\mu\varepsilon\omega^2}\right] ,$$

$$\tilde{\sigma} = \sigma\left(1-2hA_{eo}\right) .$$

Now we have

$$A_v = \alpha_{av} + i\beta_{av}$$

where

$$\alpha_{av} = A_{eo}(1+f) ,$$

$$\beta_{av} = 4\pi\mu\omega\sigma h(1-f) , \tag{6.29}$$

$$f = \frac{\left[a_0 c\, 4\pi\mu\omega h\left(\frac{\partial\sigma}{\partial\gamma}\right)\right]}{\left[A_{eo}^2+(4\pi\mu\omega h)^2\right]} .$$

The propagation vector for the v component may be written as

$$k_v = \alpha_v + \beta_v ,$$

with

$$\alpha_v = \sqrt{\frac{\mu\varepsilon'}{2}}\left\{\sqrt{1+\left(\frac{4\pi\sigma'}{\varepsilon'\omega}\right)^2}+1\right\}^{\frac{1}{2}} ,$$

$$\beta_v = \left(\frac{\omega}{c}\right)\sqrt{\frac{\mu\varepsilon'}{2}}\left\{\sqrt{1+\left(\frac{4\pi\sigma'}{\varepsilon'\omega}\right)^2}\right\}^{\frac{1}{2}} ,$$

$$\varepsilon' = \varepsilon\left[1-\frac{\left(\alpha_{av}^2-\beta_{av}^2\right)}{\mu\varepsilon\omega^2}+\left(\frac{A_{eo}f}{\mu\varepsilon\omega^2 h}\right)\right] ,$$

$$\sigma' = \sigma\left[1-\left(\frac{2a_{av}\beta_{av}}{4\pi\mu\omega\sigma}\right)-f\right] .$$

(6.30)

Now we have

$$A_b = \alpha_{ab} + i\,\beta_{ab}$$

with

$$\alpha_{ab} = A_{eo} D - 4\pi\omega\sigma h F \quad ,$$
$$\beta_{ab} = A_{eo} F + 4\pi\mu\omega\sigma h D \quad ,$$
$$D = \frac{\left(\alpha_v \alpha_e + \beta_v \beta_e\right)}{\left(\alpha_e^2 + \beta^{e^2}\right)} \quad ,$$
$$F = \frac{\left(\alpha_e \beta_v + \alpha_v \beta_e\right)}{\left(\alpha_e^2 + \beta_e^2\right)} \quad .$$

Then

$$k_b = \alpha_b + i\,\beta_b \tag{6.31}$$

where

$$\alpha_b = \left(\frac{\omega}{c}\right)\sqrt{\frac{\mu\hat{\varepsilon}}{2}}\left\{\sqrt{1 + \frac{4\pi\hat{\sigma}}{\varepsilon'\omega^2}} + 1\right\}^{\frac{1}{2}} \quad ,$$

$$\beta_b = \left(\frac{\omega}{c}\right)\sqrt{\frac{\mu\hat{\varepsilon}}{2}}\left\{\sqrt{1 + \left(\frac{4\pi\hat{\sigma}}{\hat{\varepsilon}\omega}\right)^2} - 1\right\}^{\frac{1}{2}} \quad ,$$

$$\hat{\varepsilon}' = \varepsilon\left[1 - \frac{\alpha_{av}^2 - \beta_{av}^2}{\mu\varepsilon\,\omega^2}\right] \quad ,$$

$$\hat{\sigma}' = \sigma\left[1 - \frac{2\alpha_{av}\beta_{av}}{4\pi\mu\omega\sigma} - f\right] \quad .$$

The system of equations, Eqns. (6.26) through (6.31), representing the solution is an extremely complicated system and should be put on a computer in order to fully explore all the ramifications of this solution.

6.3 Non-thermal Transmission through Media

It is the intent of this section to briefly show the effect of the solution given above and discuss how this solution may be useful in modeling electromagnetic interactions with biological systems. Therefore, consider the question of component attenuation, or how the different components of the electromagnetogravitic wave may be attenuated? A simpler question would be, "For what frequencies will the components not be attenuated at all?"

From Eqn. (6.28) we find that the electric component will pass unattenuated if σ=0. This is satisfied by two conditions. The first condition is that σ = 0 which is the classical condition of a perfect dielectric. The other condition is that

$$h = \left(\frac{1}{2 A_{eo}} \right) \tag{6.32}$$

Substituting the definition for h into Eqn. (6.32) we find, after some manipulation, that this is satisfied if

$$\left(\frac{\mu\varepsilon\,\omega^2}{A_{eo}^2} \right) - \left(\frac{\mu\varepsilon\,\omega^2}{A_{eo}^2} \right)^2 - \left(\frac{\mu\varepsilon\,\omega^2}{A_{eo}^2} \right) - 3 = 0 \quad . \tag{6.33}$$

which has only one real solution

$$\mu\varepsilon\,\omega_c^2 = 1.7971\, A_{ae}^2 \quad .$$

The complex solutions are:

$$\mu\varepsilon\,\omega^2 = (-0.8985 + \_\; i\, 1.0434)\, A_{eo} \quad .$$

Considering the real solution and assuming A_{ae}^2 to be real, we find that

$$\omega_c = \sqrt{\frac{1.7971}{\mu\varepsilon}}\, A_{eo} \quad .$$

We do not yet know the dependence of A_{eo} upon μ, ε, ω. The assumption that A_{eo} is linear in ω would mean that the relative strength of the gravitational component compared with the electric component, given by

$$V_y = -\left(\frac{A_e}{\omega} \right) E_y \quad ,$$

does not depend upon frequency in free space. In Eqn. (6.33), the assumption that

$$A_{eo} = \eta\omega \quad , \tag{6.34}$$

implies that there are no frequencies for which $\beta_e = 0$, and this would be consistent with classical theory.

If now we look at the frequencies for which $\beta_v = 0$, we find, from Eqn. (6.29), that $\beta_v = 0$ when $\sigma' = 0$, or

$$1 - \left(\frac{2\alpha_{av}\beta_{av}}{4\pi\mu\omega\sigma} \right) = f \quad . \tag{6.35}$$

Substituting for the defined quantities in Eqn. (6.35), assuming $\eta^2 << \mu\varepsilon$, and disregarding negative frequencies, we find two possible frequencies for which $\beta_v = 0$, or

$$\omega_{c1} \cong \left(\frac{1}{\eta} \right) \left[\frac{-16\pi\eta\, a_0 c \left(\frac{\partial\sigma}{\partial\gamma} \right)}{2\mu\varepsilon^2} - \left(\frac{4\pi}{\varepsilon} \right)^2 \right]^{\frac{1}{2}} , \tag{6.36}$$

and

$$\omega_{c2} \cong \left(\frac{1}{\eta} \right) \left\{ \frac{4\pi\, a_o c \left(\frac{\partial\sigma}{\partial\gamma} \right)}{2\varepsilon \left(1 - \frac{2\eta}{\mu\varepsilon} \right)} \left[2 + \left(\frac{4\pi}{\mu\varepsilon} \right) \left(1 - \frac{2\eta}{\mu\varepsilon} \right) \right] - \left(\frac{4\pi}{\varepsilon} \right)^2 \right\}^{\frac{1}{2}}$$

The magnetic component is unattenuated when $\beta_b = 0$, or when

$$4\pi\mu\omega\sigma = 2\alpha_{ab}\beta_{ab} \quad . \tag{6.37}$$

The condition specified by Eqn. (6.37) represents a seventh order polynomial in ω, therefore, the roots of this polynomial have not been sought. It may be noted though that there are up to seven possible frequencies for which the magnetic component is unattenuated.

Thus, for frequencies satisfying the conditions of Eqns. (6.35) and (6.37), the gravitational or the magnetic component respectively will experience no attenuation. Because these conditions result in polynomials in ω, then there must be frequency regions where either $\beta_v < 0$ or $\beta_b < 0$, or both β_v and β_b are negative. In these regions the gravitational and/or the magnetic component will experience an amplitude growth.

On the other hand, from Eqn. (6.32), we found that there were no frequencies for which $\beta_e < 0$ for $\sigma > 0$. This then leads to the possibility that the growth in the gravitational and/or magnetic component is at the expense of the electric component.

If then, non-thermal transmission is defined to be transmission during which none of the wave energy is deposited in the media, we find that our simple solution will support non-thermal transmission for frequencies satisfying Eqn. (6.33). For this type of transmission the energy originally in the electric component experiences an attenuation and is transferred to the gravitational and/or the magnetic components which experience a gain. The net result is the transmission of energy through the media without loss, only a change in form.

We have seen that the three fundamental postulates of the Dynamic Theory have led to the use of mass density as a fifth dimension, fundamentally independent of space and time. The five-dimensionality of the theory produced the eight differential equations describing the allowed interrelationships between the five dimensional gauge fields. Other investigations in fields allowed for fundamental particles produces the interpretation of the V field as the gravitational field.

With the interpretation of the V field came the question of how these waves might propagate given their specified interrelationships. In answer to this question we've shown that for simplified, continuous media there exist frequencies for which the electric component is attenuated while gain in the gravitational and/or magnetic components is experienced. This gives rise to the possibility of transmitting energy through the conductors where no such energy transmission is allowed by Maxwellian electromagnetism.

Even though biological systems are complex structures, is it not possible that the five-dimensional fields of the Dynamic Theory have applicability in describing radiation interaction with these systems? Is it possible that a description of non-thermal effects of radiation on biological systems may be aided through the use of the non-thermal transmission effects discussed here? A great deal of discussion these days concentrates on nonlinear approaches. The five dimensional waves provide a linear description of effects which in four dimensions would appear as nonlinear. Thus, it would seem possible to replace nonlinear four-dimensional problems with five dimensional linear ones.

6.4 Boundary Conditions

Classical work on boundary conditions of field vectors generally starts with Coulombs' Law. Here polarization of materials enter the picture. Polarization can be caused by either alignment of molecules or induced asymmetry. From these considerations the electric dipole moment is defined as

$$\bar{p} = q\bar{l}$$

where p is the electric dipole moment and l is a vector from -q to +q. The net dipole moment per unit volume is the polarization, P, of the medium. From this we get

$$\int_{vol} \overline{\Delta} \bullet \overline{P} \;\; dvol = q'$$

where q is the net polarization charge within the volume. If the density of the polarization charge is ρ'l then we have

$$q' = - \int_{vol} \rho' \, dvol$$

where the minus sign arises since by definition, the direction of the polarization vector is from negative to positive, whereas the electric field is directed from positive to negative. Thus, we arrive at

$$\overline{\Delta} \bullet \overline{P} = - \rho'$$

Now in order to write an equation like

$$\overline{\Delta} \bullet \overline{E} = 4\pi\rho$$

that is valid in a dielectric medium and account for both free charges and polarization, ρ and ρ' respectively we must write

$$\overline{\Delta} \bullet \overline{E} = 4 (\rho + \rho') \;\; , \tag{6.38}$$

or using Equation (3.53)

$$\overline{\Delta} \bullet (\overline{E} + 4\pi \, \overline{P}) = 4\pi\rho \;\; .$$

Maxwell named the quantity in parenthesis the dielectric displacement, or

$$\overline{D} = \overline{E} + 4\pi \, \overline{P} \;\; .$$

Therefore, the equation for a dielectric media becomes

$$\overline{\Delta} \bullet \overline{D} = 4\pi\rho \;\; .$$

From experiment it is found that a large class of media exhibit P proportional to E, for field strengths not too great. Thus,

$$\overline{P} = \chi_e \overline{E} \tag{6.40}$$

where x_e is the electric susceptibility of the medium.
Then

$$\overline{D} = (1 + 4\pi\ \chi_e)\overline{E}\quad .$$

The proportionality factor between D and E is called the dielectric constant and

$$\varepsilon = 1 + 4\pi\ \chi_e\quad .$$

Therefore,

$$\overline{D} = \varepsilon\,\overline{E}\quad .$$

Thus, Equation (6.39) may be written as

$$\overline{\Delta} \bullet (\varepsilon\,\overline{E}) = 4\pi\rho\quad . \tag{6.41}$$

Consider now that in the Dynamic Theory we derived the equation

$$\overline{\Delta} \bullet (\varepsilon\,\overline{E}) = 4\pi\rho - ao\frac{\partial(\varepsilon\,V_4)}{\partial\gamma}\quad . \tag{6.42}$$

Thus it may be seen by comparing Equation (6.41) with Equation (6.42), that the second term on the right hand side plays a role of gravitational polarization charge density, or

$$ao\frac{\partial(\varepsilon V4)}{\partial\gamma} = \frac{4\pi}{\varepsilon}\overline{\Delta} \bullet \overline{P}_g\quad . \tag{6.43}$$

Then we would have

$$\overline{\Delta} \bullet (\varepsilon\,\overline{E} + 4\pi\ \overline{P}_g) = 4\pi\rho\quad ,$$

or

$$\overline{\Delta} \bullet \left[(1 + 4\pi\ \chi_e)\overline{E} + 4\pi\ \overline{P}_g\right] = 4\pi\rho\quad ,$$

or

$$\overline{\Delta} \bullet \left[\overline{E} + 4\pi\ \overline{P} + 4\pi\ \overline{Pg}\right] = 4\pi\rho\quad . \tag{6.44}$$

Thus, in order to include the gravitational polarization the dielectric displacement must be given by

$$\overline{D} = \overline{E} + 4\pi \overline{P} + 4\pi \overline{P}_g \quad .$$

Now the dielectric polarization, P, is an averaging over a finite volume, thus when using the Gaussian pillbox in looking at boundary conditions it is assumed that the pillbox may contain free charges but not polarization charges. Thus, when considering boundary conditions we must look at the displacement vector D not the field strength E.

Therefore, consider the usual "Gaussian pillbox" of cross-sectional area S and thickness Δt. Let n be the unit normal to the surface S. The pillbox volume is V = SΔt and is assumed to contain free charge but no polarization charge, nor gravitational polarization (we may want to rethink this assumption concerning gravitational polarization when it is better understood.) If we integrate over the volume V we have

$$\int_v \overline{\Delta} \bullet \overline{D}\, dv = 4\pi \int_v \rho dv$$

or, by using the divergence theorem,

$$\int_s \overline{D} \bullet d\overline{a} = 4\pi \int_v \rho dv \quad .$$

The left hand side may be integrated by noting that since the normal component of D is involved there is no contribution from the sides. Thus, since the volume V can be made sufficiently small, we have

$$\overline{D}_2 \bullet \hat{n}_2 + \overline{D}_1 \bullet \hat{n}_1 = (\overline{D}_2 \bullet \hat{n}_2 - \overline{D}_1 \bullet \hat{n}_1) S = 4\pi\ \rho\ S\, \Delta t \quad .$$

If

$\lim_{\Delta t \to 0} (\rho \Delta t) = \rho s$ 57; or free surface charge density, then

$$(\overline{D}_2 - \overline{D}_1) \bullet \hat{n} = 4\pi \quad \rho s$$

(6.45)

relates the charge in the normal component of D across a boundary to the surface density of free charge on that boundary. If ρs = 0 then the normal component D is continuous across the boundary. Equation (6.41) may also be written as

$$(\varepsilon_2 \overline{E}_2 + 4\pi\ \overline{P}_{g2} - \varepsilon_1 \overline{E}_1 - 4\pi\ \overline{P}_{g1}) - n = 4\pi \quad \rho s \quad .$$

This points out the need to consider the physical meaning of the gravitational polarization but we won't go into that at this time.

The next condition that must be fulfilled at the boundary comes from

$$\bar{\Delta} x \bar{E} = \bar{O}$$

for static fields so that ∂B/∂t = 0. (This may safely be assumed since even for non-magnetostatic field the contribution by the ∂B/∂t term vanishes in using Stokes theorem).

Now construct a rectangular path which has sides Δ1 width Δt, and for which the sides parallel a segment of the bounding surface. Then by Stokes' theorem

$$\int \bar{E}.d\bar{l} = \int_s (\bar{\Delta}.\bar{E}).\hat{n}_o da = 0 \quad .$$

Thus,

$$\bar{E}.d\bar{l} = (\bar{E}_1 \cdot \hat{n}_1)\Delta 1 + (\bar{E}_2 \cdot \hat{n}_2)\Delta 1 + \textit{contribution from ends.}$$

Now,

$\hat{n}_1 = -\hat{n}_2$, 63 therefore

$$(\bar{E}_2 - \bar{E}_1).\hat{n}_2 \Delta 1 + (ends) = 0 \quad .$$

Since the contribution from the ends is proportional to Δt, the second term vanishes in the limit as Δt → 0. Thus we have

$$(\bar{E}_2 - \bar{E}_1) - \hat{n}_2 = 0 \quad ,$$
$$n_0 \cdot (\bar{E}_2 \bar{E}_1) \, x \, \hat{n} = 0 \quad .$$

Now

$$\hat{n}_2 = \hat{n}_0 \, x \, \hat{n}$$

so that

$$(\bar{E}_2 - \bar{E}_1).(\hat{n}_1 \, x \, \hat{n}) = 0$$

or

$$n_0 \cdot (\bar{E}_2 \bar{E}_1) x \hat{n} = 0 \quad .$$

But the orientation of the rectangle is arbitrary so that

$$(E_2 - E_1) \times \hat{n} = 0 \quad .$$

(6.46)

Eqn. (6.42) implies that the tangential component of E must be continuous.

For the magnetic induction field, since here

$$\overline{\Delta} \bullet \overline{B} = 0$$

which is the same as in the classical case we would have

$$(\overline{B}_2 - \overline{B}_1) \bullet \hat{n} = 0$$

or the normal component of B is continuous across the boundary.

For the tangential component we must consider the possibility of another source term as we did for

$\overline{\Delta} \bullet \overline{E}$. 73 For an Amperian loop current I, a directed loop area S, the magnetic dipole moment is defined by

$$\overline{m} = \frac{I' \overline{S}}{c} \quad .$$

Averaging over a volume we obtain the magnetization, M, which is the net dipole moment per unit volume,

$$\overline{M} = \frac{d\overline{m}}{dv} \quad .$$

From this the Amperian current density becomes

$$\overline{J} = c \; \overline{\Delta} \times \overline{M} \quad .$$

Now, in the classical case, we have, for electrostatic fields,

$$\overline{\Delta} \times \overline{B} = \frac{4\pi}{c} (\overline{J} + \overline{J}')$$

where J is the Amperian current density. Thus, we could write

$$\overline{\Delta} \times (\overline{B} - 4\pi \; \overline{M}) = \frac{4\pi}{c} \overline{J} \quad .$$

We can then define the magnetic intensity vector as

$$\overline{H} = \overline{B} - 4\pi \; \overline{M} \quad .$$

Again, experimentally, numerous materials are found such that

$$\overline{M} = \chi_m \overline{H}$$

is a good approximation for the magnetization for small fields. x_m is called the magnetic susceptibility. Thus we would have

$$\overline{B} = (1 + 4\pi\ \chi_m)\ H = \mu \overline{H}$$

where μ is the permeability of the material.

Now in the Dynamic Theory we have, when M□/Mt = 0,

$$\overline{\Delta}\ x\ (\overline{B}/\mu) = \frac{4\pi}{c}\overline{J} - a_0 \frac{(\overline{V}/\mu)}{\partial\gamma} \quad .$$

(6.47)

If we define a gravitational magnetization by

$$4\,\overline{\Delta}\ x\ \frac{\overline{M}_g}{\mu} = \frac{-\,a_0}{\mu}\ \frac{\overline{V}}{\gamma} \quad ,$$

then Eqn. (6.43) becomes

$$\overline{\Delta}\ x \left[\frac{\overline{B}}{\mu} - 4\pi\ \frac{\overline{M}_g}{\mu} \right] = \frac{4\pi}{c}\overline{J}$$

and the magnetic intensity vector should be defined by

$$\mu \overline{H} = \overline{B} - 4\pi\ \overline{M}_g$$

or

$$\overline{H} = \overline{B} - 4\pi\ (\overline{M} + \overline{M}_g) \quad .$$

(6.48)

It may be seen from Eqn. (6.44) that the gravitational magnetization adds to the Amperian magnetization and could lead to misinterpretations.

Now, in an analogous fashion to the dielectric displacement vector, the boundary condition becomes

$$(\overline{H}_2 - \overline{H}_1)\ x\ \hat{n} = \left(\frac{4\pi}{c}\right)\overline{K}$$

where K is the surface current density according to

$$\overline{K} = \lim_{\substack{\Delta t \to 0 \\ \overline{J} \to \infty}} (\overline{J}\Delta t)$$

Thus, if there is no surface current density, then

$$(\overline{H}_2 - \overline{H}_1)\, x\, \hat{n} = 0$$

or the tangential component of H must be continuous across the boundary.

Now we need to do a similar thing for V and V_4. Starting with the equation

$$\overline{\Delta}.\frac{\overline{V}}{\mu} + \frac{1}{c}\frac{\partial(\varepsilon V_4)}{\partial t} = -\frac{4\pi}{c} J_4$$

we will look at the gravitational field defined at

$$\overline{G} = \frac{\overline{V}}{\beta\mu}$$

where β is the gravitational charge-to-mass ratio. Thus, we have

$$\overline{\Delta}.\overline{G} + \frac{1}{c}\frac{\partial\left(\dfrac{\varepsilon V_4}{\beta}\right)}{\partial t} = -\frac{4\pi}{\beta c} J_4$$

where the quantity ($J_4/\beta c$) is the free gravitational mass density.

Now by using the divergence theorem we have

$$\int_s \overline{G}.\overline{d}a + \frac{1}{c}\int_v \frac{\partial\left(\dfrac{\varepsilon V_4}{\beta}\right)}{t} dv = -4\pi \int_v \left(\frac{J_4}{\beta c}\right) dv \quad .$$

By using a Gaussian pillbox again, then for a sufficiently small box

$$(\overline{G}_2 \cdot \overline{n}_2 + \overline{G}_1 \cdot \overline{n}_1) + \frac{\varepsilon S\, \Delta t}{\beta c}\frac{\partial V_4}{\partial t} = -4\pi\left(\frac{J_4}{\beta c}\right) S\Delta t \quad . \tag{6.49}$$

For free gravitational mass density to exist on the boundary, the product J_4 Δt must remain finite as t → 0. Therefore, in the limit

$$\lim_{\Delta t \to 0}\left[\frac{J_4\,\Delta t}{\beta c}\right] = m_s \quad .$$

Then Eqn. (6.45) becomes

$$(\overline{G}_2 - \overline{G}_1).\hat{n} = -4\pi \; m_s \quad .$$

Thus, if there is no free gravitational mass on the surface, the normal component of the gravitational field must be continuous.

Question: The relationship between G and V is similar to that between H and B. Is it fair then to consider a similar behavior between them? By this I mean since

$$\overline{B} = \mu \overline{H} = \overline{H} + 4\pi \; \chi_m \overline{H} = \overline{H} + 4\pi \; \overline{M}$$

then can

$$\frac{\overline{V}}{\beta} = \mu \overline{G} = \overline{G} + 4\pi \; \chi_g \overline{G} = \overline{G} + 4\pi \; \overline{N}$$

where χ_g may be called the gravitational susceptibility and N is yet unnamed?

Now lets look at

$$\overline{\Delta} \, x \, \overline{V} = -ao \frac{\partial \overline{B}}{\partial \gamma}$$

For this we construct the closed rectangular path across the boundary. Then we would have

$$\int \overline{V}.d\bar{l} = \int_s (\overline{\Delta} \, x \, \bar{v}).\bar{n}_0 \, d'a' = ao \int_s \frac{\partial}{\partial \gamma} (\overline{B}.n_0) da$$

Where S is the rectangular area $\Delta 1 \; \Delta T$ and n_o is the unit vector normal to the rectangle and lies along the boundary of the surface. Performing the line integral, we obtain

$$\int (\overline{V}.d\bar{l}) = (\overline{V}_1 \cdot \hat{n}_1) \Delta 1 + (\overline{V}_2 \cdot \hat{n}_2) \Delta 1$$
$$+ (\text{contribution from ends})$$
$$= -ao \left(\frac{\partial \overline{B}}{\partial \gamma} \right) . \hat{n}_0 \, \Delta 1 \, \Delta t$$

But since

$\hat{n}_1 = -\hat{n}_2$ 102 and the contribution from the ends is proportional to Δt we have, as $\Delta t \rightarrow 0$.

$$(V_2 - V_2).(\hat{n}_0 \, x \, \hat{n}_1) = 0$$

or

$$\hat{n}_0 \cdot \; (\overline{V}_2 - \overline{V}_1) \, x \, \hat{n} = 0 \quad .$$

But, since the orientation of the rectangle is arbitrary, then

$$(\overline{V}_2 - \overline{V}_1)\, x\, \hat{n} = 0$$

This states that the tangential component of V, the gravitational induction, must be continuous.

The boundary condition for the scalar, V_4 is that V_4 must be continuous because it is a scalar.

It perhaps should be noted that the new physical notions that appear in the foregoing could prove extremely important should one consider going into materials development.

6.5 Reflection and Refraction

First we shall consider normal incidence as shown in Figure (15)

Figure 1. EMG wave propagating in the z-direction.

Applying the boundary conditions on the tangential components we have:

$$E_0^0\, e^{ik_{4eo}{}^{\gamma_o}} - E_1^0\, e^{ik_{4E_1}{}^{\gamma_1}} = E_2^0\, e^{ik_{4e_2}{}^{\gamma_2}}$$
$$H_0^0\, e^{ik_{4bo}{}^{\gamma_o}} + H_1^0\, E^{ik_{4b_1}{}^{\gamma_1}} = H_{2_0}\, e^{ik_{4b_2}{}^{\gamma_2}}$$

and

$$V_0^0\, e^{ik_{4vo}\gamma_o} - V_1^0\, e^{ik_{4v1}\gamma_1} = V_2^0\, e^{ik_{4v2}\gamma_2}$$

or by using $\gamma_1 = \gamma_o$ we have

$$E_0^0 - E_1^0 = E_2^0\, e^{i(k_{4e2}\gamma_2 - k_{4e1}\gamma_1)}$$
$$H_0^0 - H_1^0 = H_2^0\, e^{i(k_{4b2}\gamma_2 - k_{4b1}\gamma_1)}$$

(6.50)

and

$$V_0^0 - V_1^0 = V_2^0\, e^{i(k_{4v2}\gamma_2 - k_{4v1}\gamma_1)}$$

But

$$\overline{H} = \frac{\overline{B}}{\mu} - \frac{4\pi\, \overline{M}_g}{\mu} \quad ,$$

and

$$B_y = \left(\frac{k_e c}{\omega}\right) E_x \ .$$

Therefore, if we assume for the moment that gravitational magnetization is zero, or
$\overline{M}_g = 0,$ 112 then

$$H_0^0 = \left(\frac{k_{e1} c}{\omega}\right) E_0^0 \ .$$

Note: This assumption places some, perhaps severe, restrictions upon $\partial\overline{V}/\partial\gamma$ 114 and we will have to come back and look at these, but for now it seems like a reasonable assumption to allow us to proceed with reasonable simplicity.

Thus, from the first two of Eqn. (6.50) we have

$$E_0^0 - E_1^0 = E_2^0 e^{i\Delta\gamma_e} \tag{6.51}$$

and

$$E_o^o + E_1^o = \left(\frac{k_e 2}{k_e 1}\right) E_2^o e^{i\Delta\gamma_e} .$$

Where

$$\Delta\gamma_e \equiv k_{4e2}\gamma_2 - k_{4e1}\gamma_1 \ .$$

By adding Eqn. (6.51) we find, after rearranging the terms,

$$E_2^0 = \frac{2 k_{e1} e^{-i\Delta\gamma e}}{[k_e + k_{e2}]} E_0^0 \ . \tag{6.52}$$

On the other, by subtracting Eqn. (6.51) and rearranging, we have

$$E_1^0 = \frac{(k_{e2} - k_{e1})}{[k_{e1} + k_{e2}]} \ E_0^0 \ .$$

For the solution sought in the non-thermal biological section E_z=V_4=V_z=0. Thus, if we stay with that solution we need only look at V_x, but

$$V_x + \left(\frac{-a_0 ck 4_e}{\omega}\right) E_x \ .$$

Defining

$$A_e = a_0 c k_{4e} \quad ,$$

then

$$V_x = \left(\frac{-A_e}{\omega}\right) E_x \quad .$$

Thus, the last of Eqn. (6.50) becomes

$$E_1^0 - E_0^0 = \left(\frac{-A_e}{A_e}\right) e^{i\Delta\gamma e} E_2^0 \quad .$$

(6.53)

Using Eqn. (6.52) this becomes

$$E_1^0 = \left[1 - \frac{A_{e2}}{A_{e1}} \frac{2\,k_{e1}}{(k_{e1} + k_{e2})}\right] E_0^0$$

(6.54)

Comparing Eqn. (6.54) with Eqn. (6.53) we find

$$-k_{e1} = k_{e1} - \left(\frac{A_{e2}}{A_{e1}}\right)(2\,k_{e1})$$

which is only satisfied only if

$$A_{e1} = A_{e2} \quad .$$

(6.55)

Equation (6.55) implies that the dependence of the electric field upon mass density is not influenced by the type of material there. This is a result that is a direct consequence of the assumption previously made and is further evidence that we must return to that assumption soon. For now we shall forge ahead.

We shall now consider the case where the incident wave impinges upon the boundary interface at an oblique angle θ_o. The wave is polarized so that the electric component is parallel with the interface.

For the incident wave we have

$$\left.\begin{aligned} \overline{E}_0 &= \overline{E}_0^0 e^{-i(wt-\overline{k}_{eo}\bullet\overline{r}-k_{4eo}\gamma_1)} \\ \overline{H}_0 &= \overline{H}_0^0 e^{-i(wt-\overline{k}_{b0}\bullet\overline{r}-k_{4bo}\gamma_1)} \\ &= \left(\frac{c}{\mu_1\omega}\right)\left(\overline{k}_{eo} x \overline{E}_0\right) \\ \text{and} \quad \overline{V}_0 &= \overline{V}_o^o e^{-i(wt-\overline{k}_{vo}\bullet\overline{r}-k_{4vo}\gamma_1} \\ &= \left(\frac{-A_{eo}}{\omega}\right)\overline{E}_0 \quad . \end{aligned}\right\}$$

For the reflected waves

$$\left.\begin{aligned} \overline{E}_1 &= \overline{E}_1^0 e^{-i(wt-\overline{k}_{e1}\bullet\overline{r}-k_{4e1}\gamma_1)} \\ \overline{H} &= \left(\frac{c}{\mu_1\omega}\right)\left(\overline{k}_{e1} x \overline{E}_1\right) \\ \text{and} \quad \overline{V} &= \left(\frac{-A_{e1}}{\omega}\right)\overline{E}_1 \end{aligned}\right\}$$

(6.56)

The refracted waves are given by

$$\left.\begin{aligned} \overline{E}_2 &= \overline{E}_1^0 e^{-i(wt-\overline{k}_{e2}.\overline{r}-k_{4e2}\gamma_2)} \\ \overline{H}_2 &= \left(\frac{c}{\mu_2\omega}\right)\left(\overline{k}_{e2} x \overline{E}_2\right) \\ \text{and} \quad \overline{V}_2 &= \left(\frac{-A_e}{\omega}\right)\overline{E}^2 \end{aligned}\right\}$$

(6.57)

The tangential components of □, H, and V, can be continuous across the boundary only if the phases of the field vectors are all equal at the interface.

$$\begin{aligned} \overline{k}_{eo}\bullet\overline{r}+k_{4eo}\gamma_1 &= \overline{k}_{e1}\bullet\overline{r}+k_{4e1}\gamma_1 = \overline{k}_{e2}\bullet\overline{r}+k_{4e2}\gamma_2 \\ \overline{k}_{bo}\bullet\overline{r}+k_{4bo}\gamma_1 &= \overline{k}_{b1}\bullet\overline{r}+k_{4b1}\gamma_1 = \overline{k}_{b2}\bullet\overline{r}+k_{4b2}\gamma_2 \\ \overline{k}_{vo}\bullet\overline{r}+k_{4vo}\gamma_1 &= \overline{k}_{v1}\bullet\overline{r}+k_{4v1}\gamma_1 = \overline{k}_{v2}\bullet\overline{r}+k_{4v2}\gamma_2 . \end{aligned}$$

(6.58)

For each component the propagation vectors k_{eo}, k_{e1}, and k_{e2} are coplanar, so if r is chosen to lie in the interface and in the plane of the propagation vectors, then we have,

$$k_{eo}\sin\theta_{eo} + k_{4eo}\gamma_1 = k_{e1}\sin\theta_{e1} + k_{4e1}\gamma_1 = k_{e2}\sin\theta_{e2} + k_{4e2}\gamma_2.$$

For k_{eo}-k_{e1} we find

$$\sin\theta_{e1} = \sin\theta_{eo} - \left(\frac{\gamma_1}{k_{eo}}\right)(k_{4e1} - k_{4eo}).$$

From this we find that $\sin\theta_{eo} = \sin\theta_{e1}$ if $\gamma_1 = 0$ or if $k_{4e1} = k_{4eo}$. 133 Not wanting to restrict ourselves un-necessarily by assumptions, lets continue.

For other components we have

$$\sin\theta_{b1} = \sin\theta_{bo} - \left(\frac{\gamma_1}{k_{bo}}\right)(k_{4b1} - k_{4bo})$$

and

$$\sin\theta_{v} = \sin\theta_{V1} - \left(\frac{\gamma_1}{k_{vo}}\right)(k_{4_{v_1}} - k_{4_{v_2}}).$$

Again using Eqn. (6.58) we find

$$k_{e1}\sin\theta_{e1} - k_{e2}\sin\theta_{e_2} = k_{4e_2\gamma_2} - k_{4e_1\gamma_1} \tag{6.59}$$

and

$$k_{eo}\sin\theta_{eo} - k_{e2}\sin\theta_{e_2} = k_{4e_2\gamma_2} - k_{4eo\gamma_1} \tag{6.60}$$

However, for $k_{eo} = k_{e1}$, subtracting Eqn. (6.60) from Eqn. (6.59) yields

$$\gamma_1(k_{4eo} - k_{4e1}) = 0$$

so that we have as a required result

$$k_{4eo} = k_{4e1}. \tag{6.61}$$

In a similar fashion we have

$$k_{4bo} = k_{4b1}$$

and

$$k_{4vo} = k_{4v1}$$

With this result Eqn. (6.59) becomes

$$\sin\theta_{e2} = \left(\frac{k_{e1}}{k_{e2}}\right)\sin\theta_{e1} \frac{-(k_{4e2}\gamma_2 - k_{4e1}\gamma_1)}{k_{e2}}$$

(6.62)

Similarly, for the other components we have

$$\sin\theta_{b2} = \left(\frac{k_{b1}}{k_{b2}}\right)\sin\theta_{b1} \frac{-(k_{4b2}\gamma_2 - k_{4b1}\gamma_1}{k_{b2}}$$

(6.63)

and

$$\sin\theta_{v2} = \left(\frac{k_{v1}}{k_{v2}}\right)\sin\theta_{v1} \frac{-(k_{4v2}\gamma_2 - k_{4v1}\gamma_1)}{k_{V2}}$$

(6.64)

Because of Eqn. (6.57), we must have

$$\theta_{eo} = \theta_{e1} \; ; \; \theta_{b1} = \theta_{b1} \; ; \; \theta_{vo} = \theta_{v1} \quad .$$

Now the tangential components of □, H, and V must be continuous at the interface. Therefore,

$$(\overline{E}_o + \overline{E}_1) \, x \, \hat{n} = \overline{E}_2 x\hat{n} \quad ,$$
$$(\overline{H}_o + \overline{H}_1) \, x \, \hat{n} = \overline{H}_2 \, x \, \hat{n} \quad ,$$

(6.65)

and

$$(\overline{V}_o + \overline{V}_1) \, x \, \hat{n} = \overline{V}_2 \, x\hat{n} \quad .$$

Eqn. (6.65) may be written in terms of the electric component then we would have

$$(\overline{E}_o + \overline{E}_1) x\hat{n} = \overline{E}_2 x\hat{n},$$
$$\left[(\overline{k}_{eo} \, x \, \overline{E}_o) + (\overline{k}_{e1} \, x \, \overline{E}_1)\right] x\hat{n} = \left(\frac{\mu_1}{\mu_2}\right)(\overline{k}_{e_2} \, x \, \overline{E}_2) x\hat{n},$$

(6.66)

and

$$(A_{eo}\overline{E}_o + A_{e1}\overline{E}_1) x\hat{n} = A_{e2}(\overline{E}_2 \, x\hat{n}).$$

From the first and last of Eqn. (6.66) we have

$$(A_{eo}\overline{E}_o - A_e l E_1) x\hat{n} = A_{e2}(\overline{E}_o + \overline{E}_1) x\hat{n}$$

or

$$\left[(A_{eo} - A_{e2})\overline{E}_o + (A_{e1} - A_{e2})\overline{E}_1\right] x\, \hat{n} = 0.$$

(6.67)

Since both □o and □1 are perpendicular to the plane of incidence then Eqn. (6.66) requires that

$$(A_{eo} - A_{e2})\overline{E}_o = -(A_{e1} - A_{e2})\overline{E}_1.$$

But, since $A_{eo} = A_{e1}$, this can be satisfied only if

$$A_{e1} = A_{e2}.$$

We've seen this result before.

Now suppose we expand the triple cross products in Eqn. (6.62), then

$$\left[(\hat{n}\bullet\overline{k}_{eo})\overline{E}_o - (\hat{n}\bullet\overline{E}_o)\overline{k}_{eo}\right] + \left[(\hat{n}\bullet\overline{k}_{e1})\overline{E}_1 - (n\bullet\overline{E}_1)\overline{k}_{e1}\right] = \left(\frac{\mu_1}{\mu_2}\right)\left[(\hat{n}\bullet\overline{k}_{e2})\overline{E}_2 - (\hat{n}\bullet\overline{E}_2)\overline{k}_{e2}\right]$$

All the products

$(\hat{n}\bullet\overline{E};)$ 155 vanish, and

$\hat{n}\bullet\overline{k}_{ej} = (-1)j\, k_{ej}\cos\theta_{ej};$ 156 j = 0, 1, 2, so that

$$k_{eo}\cos\theta_{eo}\overline{E}_o - k_{e1}\cos\theta_{e1}\overline{E}_1 = \left(\frac{\mu_1}{\mu_2}\right) k_{e2}\overline{E}_2.$$

If we use the fact that $\theta_{eo} = \theta_{e1}$, and rearrange the terms we arrive at

$$(E_o^o - E_{1o})\cos\theta_{eo} = \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{k_{eo}}{k_{e1}}\right)\cos\theta_{e2}\, e^{i(k_{4e2}\gamma_2 - k_{4e1}\gamma_1)}$$

(6.68)

Since the electric vectors are all parallel to the boundary surface, we must have

$$E_O^O + E_1^O = E_2^O e^{i(k_{4e2}\gamma_2 - k_{4e1}\gamma_1)}$$

(6.69)

We may combine Eqn. (6.68) and (6.69) by subtraction to eliminate E_2^o and obtain

$$E_1^0 = \frac{\left[\cos\theta_{eo} - (\mu_1 k_{e2})\cos\theta_{e2}\right]}{\left[\cos\theta_{eo} + \left(\frac{\mu_1 k_{e2}}{\mu_2 k_{e1}}\right)\cos\theta_{e2}\right]} E_0^0 \quad . \tag{6.70}$$

Eliminating E^O we have

$$E_2^0 = \frac{2\cos\theta_{eo}\, e^{i(k_{4e1}\gamma_1 - k_{4e2}\gamma_2)}}{\left[\cos\theta_{eo} + \left(\frac{\mu_1 k_{e2}}{\mu_2 k_{e1}}\right)\cos\theta_{e2}\right]} \tag{6.71}$$

Equations (6.62) and (6.63) may be used to determine the refracted angles for each component while Eqns. (6.69) and (6.70) determine the magnitude of the reflected and refracted electric field components. The magnitudes of the reflected and refracted magnetic and gravitational components may be found using Eqns. (6.56) and (6.57).

6.6 Complex Refraction Angles

In order to solve the refraction problem when one, or both, mediums at an interface are conductors then we must have a computer code capable of solving the complex angle problem. Thus, we must learn how to interpret the complex angle of refraction then how to compute it.

We may start with the equations already derived for the sine of the refraction angles, which are

$$\sin\theta_e 2 = \left(\frac{k_{e1}}{k_{e2}}\right)\sin\theta_{e1} - \frac{(k_{4e2}\gamma_2 - k_{4e1}\gamma_1)}{k_{e2}} \quad , \tag{6.62}$$

$$\sin\theta_{b2} = \left(\frac{k_{b1}}{k_{b2}}\right)\sin\theta_{b1} - \frac{(k_{4b2}\gamma_2 - k_{4b1}\gamma_1)}{k_{b2}} \quad , \tag{6.63}$$

and

$$\sin\theta_{v2} = \left(\frac{k_{v1}}{k_{v2}}\right)\sin\theta_{v1} - \frac{(k_{4v2}\gamma_2 - k_{4v1}\gamma_1)}{k_{v2}} \quad . \tag{6.64}$$

With the realization that one or more of the k's in Eqns. (6.62), (6.63), or (6.64) may be complex then one must consider the right-hand-side to be complex. Thus, we have the situation that $\sin\theta_2$ for each component, would be complex and, therefore, we must consider θ_2 to be complex. Thus consider the case where

$$\sin\theta_2 = x + iy \tag{6.72}$$

with

$$\theta_2 = \gamma_2 + i\gamma_2 \quad .$$

Then, by trigometric identity, we may write

$$\sin\theta_2 = \sin\alpha_2 \cosh\beta_2 + i\, cosa\alpha_2 \sin h\,\beta_2 \quad . \tag{6.73}$$

Equating Eqns. (6.72) and (6.73), we find

$$\sin\alpha_2 \cosh\beta_2 + i\cos\alpha_2 \sin h\,\beta_2 = x + iy \quad . \tag{6.74}$$

From Eqn. (6.74) we find that

$$\sin\alpha_2 = \frac{x}{\cosh\beta_2}$$

and

$$y = \cos\alpha_2 \sinh\beta_2 \quad . \tag{6.75}$$

Now α_2 is a real angle and the expression for $\sin\alpha_2$ in Eqn. (6.75) reduces to the usual expression for the sine of the refraction angle. Thus, we shall take α_2 to be the angle of refraction and it is given by Eqn. (6.75).

We must now learn how to find α_2 and β_2 given x and y. We may start by rewriting Eqn. (6.75) as

$$\sin\alpha_2 = \frac{x}{\cos h\,\beta_2} \tag{6.76}$$

and

$$\cos\alpha_2 = \frac{y}{\sin h\,\beta_2} \quad .$$

Now using Eqn. (6.76) the trigometric identity

$$1 = \sin^2\theta + \cos^2\theta$$

becomes

$$1 = \left(\frac{x}{\cosh\beta_2}\right)^2 + \left(\frac{y}{\sin h\,\beta_2}\right)^2$$

or

$$1 = \left[\frac{2x}{e^{\beta^2} + e^{-\beta^2}}\right]^2 + \left[\frac{2y}{e^{\beta^2} - e^{-\beta^2}}\right]^2$$

(6.77)

when $\cosh\beta_2$ and $\sinh\beta_2$ are written in terms of exponentials.

Now suppose we define

$$w = \cosh(2\beta_2) = \frac{e^2\beta^2 + e^{-2}\beta^2}{2}$$

(6.78)

where w ∃ 1 always. Then Eqn. (6.77) becomes

$$1 = \frac{4x^2}{(e^{2\beta^2} + e^{-2\beta^2} + 2)} + \frac{4y^2}{(e^{2\beta^2} + e^{-2\beta^2} - 2)}$$

or

$$1 = \left(\frac{2x^2}{w+1}\right) + \left(\frac{2y^2}{w-1}\right)$$

This may be rewritten as

$$w^2 - 2w(x^2 + y^2) + 2(x^2 - y^2) - 1 = 0 \quad .$$

(6.79)

Equation (6.79) is a quadratic equation in w which has the solutions

$$w = (x^2 + y^2) \pm \sqrt{(x^2 + y^2 - 1)^2 + 4y^2} \quad .$$

(6.80)

The expression under the radical is non-negative (as may be shown by a lengthy procedure) so w is real, which must be the case from Eqn. (6.78).

Consider three cases:

<u>Case A:</u> $x^2 + y^2 > 1$

Let $x^2 + y^2 = 1 + \delta$ where δ , 0, then Eqn. (6.80) becomes

$$w = 1 - \varepsilon \pm \sqrt{\varepsilon^2 + 4y^2} \quad .$$

If y = 0, then because w ∃ 1, the positive sign must be used. If y = 0, then there is no need for Eqn. (6.80).

<u>Case B:</u> $x^2 + y^2 + 1$

Let $x^2 + y^2 - 1 + \delta$ where δ , 0, then Eqn. (6.80) becomes

$$w = 1 - \varepsilon \pm \sqrt{\delta^2 + 4y^2} \quad .$$

Again, because w ∃ 1, clearly the upper sign must be chosen.

<u>Case C:</u> $x^2 + y^2 = 1$

Equation (6.76) now becomes

$$w = 1 \pm 2y$$

If y , 0, the upper sign must be used. If y + 0, there is no need for Eqn. (6.80).

From Eqn. (6.80) and the above logic on how to choose the proper sign we may obtain w from x and y. Then from w we find β_2 using Eqn. (6.78) or

$$\beta_2 = \frac{\text{arc}\, cosh w}{2}$$

(6.81)

then by Eqn. (6.72) we may find α_2 from

$$\alpha_2 = \text{arc}\sin\left(\frac{x}{\cosh\beta_2}\right) \quad .$$

(6.82)

Thus Eqn.s (6.80), (6.81) and (6.82) give us α_2 and β_2 for any x and y. The α_2 and β_2 must be checked against Eqn.s (6.75) and (6.76) to resolve any ambiguities.

6.7 Assumptions and Wave Solutions

In order to try to bring to light the significance of the variation of material properties with respect to changes in the mass density in the attempt to obtain wave solutions let us consider each assumption that must be made. To do this let us begin with the form of the trial solution.

The classically assumed form for a trial solution is

$$\exp\left[-i(wt-kx)\right]$$

for plane wave propagation in the x-direction. In as far as it may be possible we would like to stay with a similar form. Thus suppose we try the form

$$\exp\left[-i(wt-kx-k_4\gamma)\right]$$

Once the form of the trial solution is chosen then one can look at the eigenvalues of the differential operations since the trial form is the exponential form. Consider the partial derivative with respect to x

$$\frac{\partial}{\partial x}\{\exp[-i(wt-kx-k_4\gamma)]\}$$
$$=i[\frac{\partial}{\partial x}(kx-k_4\gamma)]\exp[-i(wt-kx-k_4\gamma)] \quad ,$$

if we assume that

$$\frac{\partial w}{\partial x}=0$$

Since we desire to consider how waves of a certain frequency propagate it seems appropriate to adopt this assumption.

Next, consider the eigenvalue of the differention

$$\frac{\partial}{\partial x}\rightarrow i\frac{\partial}{\partial x}(kx+k_4\gamma)$$
$$=i\{x\frac{\partial k}{\partial x}+k+\gamma\frac{\partial k_4}{\partial x}+k_4\frac{\partial\gamma}{\partial x}\}$$

There is nothing thus far in our considerations that forces us to consider only those cases for which the phase is strictly linear in x, or where the density constant cannot depend upon space. In the classical case I believe one can use the Maxwell equations, or the wave equations coming from them, to show that the phase must be linear in x. The same may prove true for these five-dimensional waves but it has not yet been proven. For the sake of simplicity let's make the same assumption here and hold in abeyance any attempt to prove a linear relationship. Therefore, lets assume

$$\frac{\partial k}{\partial x} = 0$$

With regard to k_4 we have no precedence set by classical theory and with no real feeling for the physical interpretation of k_4 we are left to our own devices. For the moment suppose we make no assumptions regarding the dependence of k_4 upon space, but we might assume isotropy is mass density. Thus, our eigenvalue for the space differential operator we have

$$\frac{\partial}{\partial x} \rightarrow i(k + \gamma \frac{\partial k_4}{\partial x})$$

Now let's look at the mass density differential operator, or

$$\frac{\partial}{\partial \gamma}\{\exp[-i(\omega t - kx - k_4\gamma)]\}$$
$$= i\left[\frac{\partial}{\partial \gamma}(kx + k_4\gamma)\right]\exp[-i(\omega t - kx - k_4\gamma)]$$

if we assume

$$\frac{\partial \omega}{\partial \gamma} = 0 \quad . \tag{6.83}$$

The assumption that the frequency should not depend upon the mass density seems justifiable since we want to determine how a wave of a certain frequency will propagate. Therefore, we want to control the frequency. We should not, however, allow this desire to lock us into this assumption.

Given the assumption Eqn. (6.83) we have

$$\frac{\partial}{\partial \gamma}(kx + k_4\gamma) = \frac{x\partial k}{\partial \gamma} + \frac{\gamma\partial k_4}{\partial \gamma} + k_4 \quad .$$

By analogy with the classical result that the phase of the wave depends only linearly upon x it seems a fair assumption that we may simplify some by assuming that

$$\frac{\partial k_4}{\partial \gamma} = 0$$

We have no real justification for this assumption at the moment though. However, with this assumption our mass density operator becomes

$$\frac{\partial}{\partial \gamma} \rightarrow i(x\frac{\partial k}{\partial \gamma} + k_4) \quad .$$

If we consider the potential variation of k with respect to mass density then we run into the dependence of k upon μ and σ and whether these material properties depend upon mass density. If they do, as experiment tends to show then

$$\frac{\partial k}{\partial \gamma} = \left(\frac{\partial k}{\partial \varepsilon}\right)\left(\frac{\partial \varepsilon}{\partial \gamma}\right) + \left(\frac{\partial k}{\partial \sigma}\right)\left(\frac{\partial \sigma}{\partial \gamma}\right) \quad .$$

Since we desire to retain the correspondence to experiment we shall make no assumptions concerning the dependency of k upon mass density.

The remaining operator is the time operator. For this we have

$$\frac{\partial}{\partial t}\{\exp|-i(\omega t - kx - k_4\gamma)|\} = -i\omega \exp|-i(\omega t - kx - k_4\gamma)|$$

if we assume that

$$\frac{\partial k}{\partial t} = 0$$

and

$$\frac{\partial k_4}{\partial t} = 0 \quad .$$

From the point of view that both k and k_4 are determined by material properties then these assumptions appear appropriate for static materials. Thus the eigenvalue of the time differential operator is

$$\frac{\partial}{\partial t} \rightarrow -i\omega \quad .$$

We have now chosen a general form for the solution we will seek. But there are several potential components to the wave. For example, there are the electric transverse, magnetic transverse, gravitational transverse, electric longitudinal, and the gravitational longitudinal components. In addition there is the scalar wave component, the gravitational potential.

In the classical case it may be shown that the propagation constant may be the same for both the electric and magnetic components. That is not so with these more complex five-dimensional waves. Thus, we should allow for the possibility that each component may have a different propagator. With this in mind we will try to find wave solutions with the following trial forms.

$$\overline{E} = \overline{E_O} \exp[-i(wt - k_e x - k_{4e}\gamma)]$$
$$\overline{B} = \overline{B_0} \exp[-i(wt - k_b x - k_{4b}\gamma)]$$
$$\overline{V} = \overline{V_O} \exp[-i(wt - k_v x - k_{4v}\gamma)]$$
$$V_4 = V_{4O} \exp[-i(wt - k_4 x - k_{44}\gamma)] \quad .$$

By using these four forms which constitute our trial solution we may find that two, or all, of the k's must be the same but we aren't forcing them to equality prematurely.

Now lets put our trial solutions into the field equations. Lets start with the field equation

$$\left(\frac{1}{c}\right)\frac{\partial \overline{B}}{\partial t} + \overline{\Delta} x \overline{E} = 0 \quad .$$

(3.15b)

The x and y components of this equation require that

$$B_x = B_y = 0 \quad .$$

For the z component we have, after simplifying,

$$B_z = \left(\frac{c}{w}\right)\left(k_e + \gamma \frac{\partial k_e}{\partial x}\right) E_y \quad .$$

(6.84)

The second field equation is

$$\overline{\Delta} x \overline{V} + a_O \frac{\partial \overline{B}}{\partial \gamma} = 0 \quad .$$

(3.15f)

The x and y components require that

$$V_z = 0 \quad .$$

However, the z component gives us

$$V_y = \left(\frac{-a_0 c}{w}\right)\left(k_{4b} + x \frac{\partial k_b}{\partial \Gamma}\right)\left[\frac{k_e + \gamma \dfrac{\partial k_{4e}}{\partial x}}{k_v + \gamma \dfrac{\partial k_{4V}}{\partial x}}\right] E_y \quad .$$

(6.85)

Now look at the field equation

$$\Delta \bullet (\varepsilon \overline{E}) = 4\pi\rho - a_O \frac{\partial(\varepsilon V_4)}{\partial\gamma} \quad .$$

(3.15d)

We see that we face more assumptions. The first one concerns whether or not ε varies with space. The classical assumption seems appropriate here also. That is, if the medium is isotropic then

$$\frac{\partial\varepsilon}{\partial x} = 0 \quad .$$

The second assumption is that there are no free charges so that

$$\rho = 0 \quad .$$

The third place for a possible assumption resides in the possible dependence of ε upon mass density. However, experiment indicates that ε does vary with changes in mass density. Therefore, it seems inappropriate to assume differently.

Thus, Eqn. (3.14d) requires that

$$V4 = \left(\frac{-1}{a_O}\right) \frac{(k_e + \gamma \frac{\partial k_{4e}}{\partial x})Ex}{-\left(\frac{1}{i\varepsilon}\right)\frac{\partial\varepsilon}{\partial\gamma} + \frac{\partial k_4}{\partial\gamma} + k_{44}]} \quad . (6.86)$$

From Eqn. (6.82) we see that the gravitational potential, V_4, depends only upon the longitudinal electric field component.

The fourth field equation is

$$\left(\frac{1}{c}\right)\frac{\partial\overline{V}}{\partial t} + \Delta V_4 = a_0 \frac{\partial\overline{E}}{\partial\gamma} \quad .$$

(3.15g)

The x component of this vector equation requires

$$V_x = \left(\frac{-c}{w}\right)\{ \frac{(k_4 + \gamma \frac{\partial k_{44}}{\partial x})(k_e + \gamma \frac{\partial k_{4e}}{\partial x})}{\partial k_4}$$

$$a_O | k_{44_x} \frac{\partial k_4}{\partial\gamma} - \left(\frac{i}{\varepsilon}\right)\frac{\partial\varepsilon}{\partial\gamma} |$$

$$+ a_O (k_{4e} + x \frac{\partial k_e}{\partial\gamma} \} Ex \quad .$$

(6.88)

From the y component of Eqn. (3.15g) we find

$$V_y = \left(\frac{-a_0 c}{w}\right)\left(k_{4e} + x\frac{\partial k_e}{\partial \gamma}\right) Ey \quad .$$

(6.88)

If we compare Eqn. (6.88) with Eqn. (6.85) we must have

$$\left(k_v + \gamma\frac{\partial k_{4v}}{\partial x}\right)\left(k_{4e} + x\frac{\partial k_e}{\partial \gamma}\right) = \left(k_e + \frac{\partial k_{4e}}{\partial x}\right)\left(k_{4b} + \frac{\partial k_b}{\partial \gamma}\right) \quad .$$

(6.89)

The z component of Eqn. (3.15g) is an identity, thus we can turn to the fifth field equation, which is

$$\overline{\Delta} \bullet \overline{V} + \left(\frac{\mu\varepsilon}{c}\right)\frac{\partial V_4}{\partial t} = \frac{-4\pi\mu}{c} \quad J_4 \quad .$$

(3.15h)

The usual conductivity assumptions seem appropriate here and are taken as

$$\overline{J} = \sigma \overline{E} \quad ,and \quad J_4 = \sigma_4 V_4$$

(6.90)

Therefore, Eqn. (3.15h) becomes the indicial relation

$$c^2\left(k_v + \gamma\frac{\partial k_{4v}}{\partial x}\right)\left\{\left(k_4 + \gamma\frac{\partial k_{44}}{\partial x}\right)\left[k_e + \gamma\frac{\partial k_{4e}}{\partial x} + a_0^2\left(k_{4e} + x\frac{\partial k_e}{\partial x}\right)\right]\right.$$
$$\left.\left[k_{44} + x\frac{\partial k_4}{\partial \gamma} - \left(\frac{i}{e}\frac{\partial \varepsilon}{\partial \gamma}\right)\right]\right\} = (\mu\varepsilon\,\omega^2 + i4\pi\mu\omega\sigma\,4)\left(k_e + \gamma\frac{\partial k_{4e}}{\partial x}\right).$$

(6.91)

when Eqns. (6.86) and (6.88) are used.

The sixth field equation is

$$\overline{\Delta} x \overline{B} - \left(\frac{\mu\varepsilon}{c}\right)\frac{\partial \overline{E}}{\partial t} = \left(\frac{4\pi\mu}{c}\right)\overline{J} - a_O\frac{\partial \overline{V}}{\partial \gamma} \quad .$$

(3.15c)

The x component of this equation produces the indicial relation

$$\mu\varepsilon\,\omega^2 + i4\pi\mu\omega\sigma = c^2\left(k_{4v} + x\frac{\partial k_v}{\partial \gamma}\right)\left\{\frac{\left(k_4 + \gamma\,\frac{\partial k_{44}}{\partial x}\right)\left(k_e + \gamma\,\frac{\partial k_{4e}}{\partial x}\right)}{\left[k_{44} + x\frac{\partial k_4}{\partial \gamma} - \left(\frac{i}{\varepsilon}\frac{\partial \varepsilon}{\partial \gamma}\right)\right]} + a_O^2\left(k_{4e} + x\frac{\partial k_e}{\partial \gamma}\right)\right\} \ .$$

(6.92)

The z component of Eqn. (3.15c) is an identity but the y component gives us another indicial relation in

$$\mu\varepsilon\, w^2 + i4\pi\mu w\sigma = (k_e + \gamma\,\frac{\partial k_{4e}}{\partial x})(k_b + \gamma\,\frac{\partial k_{4b}}{\partial x}) + a_O^2 c^2 (k_{4v} + x\frac{\partial k_v}{\partial \gamma})(k_{4e} + x\frac{\partial k_e}{\partial \gamma})$$

(6.93)

The seventh field equation

$$\overline{\Delta}.\overline{B} = 0 \ .$$

(3.15a)

is an identity since $B_x = 0$. However, the last remaining field equation

$$0 = \frac{\partial \rho}{\partial t} + \overline{\Delta} \bullet \overline{J} + a_O\frac{\partial J_4}{\partial \gamma}$$

(3.15e)

is not satisfied identically. Rather, it requires

$$\sigma \frac{\partial E_x}{\partial x} = -a_O \frac{\partial(\sigma_4 V_4)}{\partial \gamma}$$

(6.94)

if we assume

$$\frac{\partial \sigma}{\partial x} = 0 \quad .$$

We are once more in a dilemma created by our ignorance. We don't know what σ_4 is other than a "gravitational conductivity". Thus, it would appear that we cannot assume that it is independent of the mass density. Thus Eqn. (6.94) may be put into the form

$$(\sigma - \sigma_4)\left(k_{44} + x \frac{\partial k_4}{\partial \gamma} \right) = i\left[\left(\frac{\sigma}{\varepsilon} \right)\left(\frac{\partial \varepsilon}{\partial \gamma} \right) - \frac{\partial \sigma 4}{\partial \gamma} \right] .$$

(6.95)

This is an extremely curious equation. It relates the electrical conductivity to the gravitational conductivity and includes in this relation the variation of the dielectric constant with respect to changes in the mass density.

In the absence of any knowledge about σ_4, the gravitational conductivity, let us assume that there is a linear relationship between it and the mass density. The only justification for this assumption comes from the association of gravitational mass to gravitational conductivity and also to inertial mass and hence mass density. Regardless of our lack of knowledge lets assume

$$\sigma_4 = \eta\gamma \ .$$

Further, in attempting further simplification lets assume

$$\sigma = \nu \sigma_4 \quad .$$

Using these assumptions we may rewrite Eqn. (6.95) as

$$k_{44} + x\left(\frac{\partial k_4}{\partial \gamma} \right) = \left(\frac{i}{1-\nu} \right)\left\{ \frac{\partial}{\partial \gamma}\left[\ln\left(\frac{\varepsilon}{\sigma \nu} \right) \right] \right\}$$

(6.96)

The right hand side of Eqn. (6.96) is in terms of quantities which should be available through experimentation, thereby giving us a differential equation in the two unknowns k_{44} and k_4.

We now have a system of equations given by the following numbered equations: (6.84), (6.86), (6.87), (6.88), (6.89), (6.91), (6.92), (6.93), and (6.96). These equations give the field components in terms of E_y and E_x. However, the eight field equations cannot determine a relationship between the longitudinal and transverse components. This is left for the energy-momentum tensor to determine.

In order to attempt a reduction of these equations we might try the simplifying assumptions:

$$\frac{\partial k_{4e}}{\partial x} = 0 \quad ,$$

$$\frac{\partial k_{4b}}{\partial x} = 0 \quad ,$$

and

$$\frac{\partial k_{4v}}{\partial x} = 0 \quad .$$

Using these assumptions let us now look at the requirements that come from the interchangeability of substitution and differentiation. For instance,

$$\frac{\partial B_z}{\partial x} = ik_b \left(\frac{k_e c}{\omega} \right) E_y \quad ,$$

when differentiation is taken first. On the other hand if we take the substitution first then we get

$$\frac{\partial B_z}{\partial x} = i\, k_e \left(\frac{k_e c}{\omega} \right) E_y \quad .$$

By comparing these two we see that

$$k_b = k_e \quad .$$

When we compare the two expressions for the partial derivative of B_z with respect to the mass density we find

$$\frac{\partial (\ln k_e)}{\partial \gamma} = i(k_{4b} - k_{4e}) \quad .$$

Turning next to the transverse gravitational component, the partial derivative with respect to x requires that

$$\frac{\partial k_e}{\partial \gamma} = 0 \quad . \tag{6.97}$$

This sets up something of a dilemma since k_e should depend upon both ε and σ. These in turn depend upon γ. Therefore, Eqn. (6.97) is a result that does not seem to correspond to experiment. Thus, our choice of simplifying assumptions appears too restrictive. But which assumption is the one that must be relaxed? An investigation into the necessity of the assumptions made seems required prior to making advancement toward the solutions of the electromagnetogravitic wave equations.

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