Explaining Earth Flyby Anomalies

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Abstract: A recent article in Physics Today discussed the anomalous velocities reported for satellites in near Earth flyby situations and presented a phenomenological formula that fits the anomalies. The authors then note that a phenomenological formula is not an explanation, but does raise some questions. They ask why the speed of light appears in the formula. How does the declination produce a physical effect? A five dimensional theory of space-time-mass provides the answer to these questions and support for the phenomenological formula. Therein lays the explanation for the anomaly.

Introduction

Flyby anomalies have been recognized for decades and many good references may be found. What is to be considered here is the phenomenological formula

$$\frac{\Delta v_\infty}{v} = K\left(\cos\delta_i - \cos\delta_f\right)$$
$$K = \frac{2\omega_e R_e}{c} = 3.099x10^{-6} \tag{1}$$

In Equation (1) the angles are the initial and final declination angles. These angles appearing in the formula prompted the authors to ask about the potential of declination producing a physical effect.

Theoretical Contribution of the Dynamic Theory

The Dynamic Theory, due to its five dimensional basis, predicts an inductive coupling between the gravitational and the electromagnetic fields. This inductive coupling was shown to be

$$\beta = \sqrt{2\pi\varepsilon_o G} \tag{2}$$

where ε_o is the dielectric constant and G is the gravitational constant. It has been shown that this coupling predicts the magnetic moment of the Earth as

$$\mu = \left(\frac{q_{eff}}{2M}\right) I\omega_e$$
$$q_{eff} = \beta M \tag{3}$$

where M is the mass of the Earth, I is the Earth's moment of inertia and q_{eff} is the effective electric charge due to the mass of the Earth.

Magnetic Force on Flybys

The magnetic force on a satellite with velocity, $\overline{v}$ is given by

$$\overline{f} = q\left(\overline{v}\times\overline{B}\right). \tag{4}$$

The magnetic field associated with a magnetic moment of a sphere is given by

$$\overline{B} = \frac{\mu_o}{4\pi} \frac{\left[3\overline{r}\left(\overline{\mu} \bullet \overline{r}\right) - \overline{\mu} r^2\right]}{r^5}. \qquad (5)$$

Using Equation (3) the magnetic moment becomes

$$\overline{\mu} = \left(\frac{q_{eff}}{2M}\right)\overline{I}\,\omega_e = \frac{\beta}{2}\frac{2MR^2}{5}\hat{\mu}\omega_e = \frac{\beta MR^2\omega_e}{5}\hat{\mu}. \qquad (6)$$

To find a change in velocity one may look at the change of momentum due to an impulse, or

$$\Delta\overline{v} = \frac{\Delta\overline{p}}{m} = \int \frac{q}{m}\left(\overline{v} \times \overline{B}\right)dt. \qquad (7)$$

The effective charge for the satellite is

$$q_{eff} = \beta m. \qquad (8)$$

Using Equations (5), (6) and (8) in Equation (7) obtains

$$\begin{aligned}
\Delta\overline{v} &= \int \beta \frac{\mu_o}{4\pi r^5}\frac{\beta MR^2\omega_e}{5}\left(\overline{v} \times \left[3\overline{r}\left(\hat{\mu} \bullet \overline{r}\right) - \hat{\mu} r^2\right]\right)dt \\
&= \frac{\beta^2 M\mu_o R^2\omega_e}{20\pi}\int \frac{1}{r^3}\left(\overline{v} \times \left[3\hat{r}\left(\hat{\mu} \bullet \hat{r}\right) - \hat{\mu}\right]\right)dt \\
&= \frac{2\pi\varepsilon_o\mu_o GMR^2\omega_e}{20\pi}\int \frac{1}{r^3}\left(\overline{v} \times \left[3\hat{r}\left(\hat{\mu} \bullet \hat{r}\right) - \hat{\mu}\right]\right)dt \\
&= \frac{2GMR^2\omega_e}{20c}\int \frac{1}{r^3}\left(\overline{v} \times \left[3\hat{r}\left(\hat{\mu} \bullet \hat{r}\right) - \hat{\mu}\right]\right)dt
\end{aligned}. \qquad (9)$$

It should be noted that the speed of light appears in the denominator because the dielectric constant of the charge to mass ratio, β, combines with the magnetic permeability.

Change of Satellite Velocity by Magnetic Field

Satellites in hyperbolic orbits have the radial position in the orbital plane given by

$$r = \frac{a\left(\varepsilon^2 - 1\right)}{\left(1 + \varepsilon\cos\phi\right)} \qquad (10)$$

where

$$a = \left|\frac{K}{2E}\right| \quad \text{where} \quad K = -GMm \qquad (11)$$

and

$$\varepsilon = \frac{1}{\cos\alpha} \quad \text{where} \quad \alpha \equiv \frac{\pi - \Theta}{2} \qquad (12)$$

with Θ being the deflection angle. The angular momentum is found to be given by

$$L = mr^2\dot{\phi}. \qquad (13)$$

Equation (10) may be differentiated with respect to time to get the radial component of the velocity as

$$\dot{r} = \frac{dr}{dt} = \frac{\varepsilon r^2\sin\phi}{a\left(\varepsilon^2 - 1\right)}\dot{\phi} = \frac{\varepsilon r^2\sin\phi}{a\left(\varepsilon^2 - 1\right)}\frac{L}{mr^2} = \frac{\varepsilon L\sin\phi}{ma\left(\varepsilon^2 - 1\right)} \qquad (14)$$

while the azimuthial component of the velocity is

$$v_\phi = r\dot{\phi} = \frac{L}{mr}. \tag{15}$$

Therefore, the total velocity in the orbital plane is

$$\begin{aligned}
\overline{v} &= \frac{\varepsilon L \sin\phi}{ma\left(\varepsilon^2 - 1\right)}\hat{r} + \frac{L}{mr}\hat{\phi} + 0\hat{\theta} \\
&= \frac{\varepsilon L \sin\phi}{ma\left(\varepsilon^2 - 1\right)}\hat{r} + \frac{L\left[1+\varepsilon\cos\phi\right]}{ma\left(\varepsilon^2 - 1\right)}\hat{\phi} + 0\hat{\theta} \\
&= \frac{L}{ma\left(\varepsilon^2 - 1\right)}\left\{\varepsilon\sin\phi\hat{r} + \left[1+\varepsilon\cos\phi\right]\hat{\phi} + 0\hat{\theta}\right\} \qquad . \qquad (16) \\
&= \frac{L\sqrt{\varepsilon^2 + 1 + 2\varepsilon\cos\phi}}{ma\left(\varepsilon^2 - 1\right)}\left\{\frac{\varepsilon\sin\phi}{\sqrt{\varepsilon^2 + 1 + 2\varepsilon\cos\phi}}\hat{r} + \frac{\left[1+\varepsilon\cos\phi\right]}{\sqrt{\varepsilon^2 + 1 + 2\varepsilon\cos\phi}}\hat{\phi} + 0\hat{\theta}\right\} \\
&= \frac{L\sqrt{\varepsilon^2 + 1 + 2\varepsilon\cos\phi}}{ma\left(\varepsilon^2 - 1\right)}\hat{v}
\end{aligned}$$

Now we may substitute Equation (16) into Equation (9) to get

$$\begin{aligned}
\Delta\overline{v} &= \frac{2GMR^2\omega_e}{20c}\int\frac{1}{r^3}\left(\frac{L\sqrt{\varepsilon^2 + 1 + 2\varepsilon\cos\phi}}{ma\left(\varepsilon^2 - 1\right)}\hat{v}\times\left[3\hat{r}\left(\hat{\mu}\bullet\hat{r}\right) - \hat{\mu}\right]\right)dt \\
&= \frac{2GMR^2\omega_e}{20c}\frac{L}{ma\left(\varepsilon^2 - 1\right)}\frac{mr^2}{L}\int\frac{\sqrt{\varepsilon^2 + 1 + 2\varepsilon\cos\phi}}{r^3}\left(\hat{v}\times\left[3\hat{r}\left(\hat{\mu}\bullet\hat{r}\right) - \hat{\mu}\right]\right)d\phi \qquad (17) \\
&= \frac{GMR^2\omega_e}{10ca^2\left(\varepsilon^2 - 1\right)^2}\int\sqrt{\varepsilon^2 + 1 + 2\varepsilon\cos\phi}\left[1+\varepsilon\cos\phi\right]\left(\hat{v}\times\left[3\hat{r}\left(\hat{\mu}\bullet\hat{r}\right) - \hat{\mu}\right]\right)d\phi
\end{aligned}$$

Conclusions

Equation (17) has the qualitative features of the phenomenological formula of Equation (1) in that it has the same coefficient that depends upon the Earth's rotation rate and the speed of light. The speed of light enters into the equation because the effective charge of both the masses is used. The vector products left in the brackets show that the final value of the change in velocity depends upon the angle of the orbit with respect to the Earth's magnetic moment. Since the Earth's magnetic North pole is nearly the normal to the equatorial plane, the dot product and the cross product may be seen to vanish for a satellite whose orbit is strictly in the equatorial plane. This argues that the satellite must have an orbit with some inclination in order to show a change in velocity. Equation (17) shows that for various orbits the change in velocity might be positive or negative as has been measured. Also, since R is a major fraction of r the overall magnitude of the bracketed portion of Equation (16) will lie within the zero to 2 value of the difference of declinations in Equation (1).

Only an integration of an actual satellite path will show final validity of the ability of Dynamic Theory to predict the anomalous velocities measured, however, the correct qualitative features are there and the quantitative value appears to be within the ballpark.